THE DIMENSION OF SPLINE SPACE \( S^1_3(\triangle) \) ON

A TYPE OF TRIANGULATION*

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(Paper from ZHANG Xiang-wei, Member of
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**Abstract:** A harmonic condition that can distinguish whether the dimension of spline space \( S^1_3(\triangle) \) depends on the geometrical character of triangulation is presented, then on a type of general triangulation the dimension is got.

**Key words:** triangulation; spline space; harmonic condition

**CLC number:** O189.12 **Document code:** A

**Introduction**

Whether the dimension of spline space \( S^1_3(\triangle) \) depends on the geometrical character of triangulation has not been solved\(^{[1]}\). In paper \([2]\), the stratified triangulation on which the dimension of \( S^1_3(\triangle) \) does not depend on the geometrical character of triangulation is presented. We analyzed the topological characters of triangulation and constructed a type of more general triangulation on which the dimension of \( S^1_3(\triangle) \) does not depend on the geometrical character of triangulation.

**Definition 1** Let \( \Omega \) be a single connected polygon, \( \triangle = \{ T_i \}_{i=1}^r \) is triangulation of \( \bar{\Omega} \). All \( 0 \leq \mu \leq k - 1 \), the spline space is

\[
S^\mu_k(\triangle) = \{ S \in C^\mu(\Omega) \mid S \mid_{T_i} \in H_k \}, \quad i = 1, 2, \ldots, r,
\]

where \( H_k \) is polynomial of degree \( k \) in two elements.

Distinctly, there are two cases of the vertex of \( T_i \): \(^1^\circ\) The vertex is on the edge of \( \Omega \). \(^2^\circ\) The vertex is in \( \Omega \). If the vertex is in \( \Omega \), this vertex is called edge inner point of triangulation or inner triangulation point. If this vertex is on the edge of \( \Omega \), it is called edge triangular point, and then there are two cases of the edges of \( T_i \): \(^1^\circ\) There is a vertice of \( T_i \) on the edge of \( \Omega \) at least. \(^2^\circ\) All

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* Received date: 1999-06-26; Revised date: 2000-04-25

**Foundation item:** the National Natural Science Foundation of China(59975057)

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of the vertices of the edge of $T_i$ are in inner of $\Omega$. So it is called triangular line of edge if there is an unique point on the edge, the second one is called inner triangular line, the standard triangulation of $\Omega$ is called a field where there is only one inner point of triangulation$^{[3]}$.

**Lemma 1**  Let $\triangle$ be a triangulation, $E$ be the number of lines of $\triangle$, and $V$ be the number of points of $\triangle$. If $E$, $V$ is finite, then there is an inner point $O$ which is public point of two triangular lines of the edge at least.

**Proof**  Supposing $O_1$ is an inner triangular point which connects only one edge point $A$. Since $\triangle$ is triangulation of $\Omega$. $O_1$ belongs to a field $\bar{O}_1$, that is, there are two inner triangulation point $O_{11}$, $O_{12}$, which connect edge point, to connect with $O_1$, as shown in Fig.1.

Let $O_{11}$ connect only one edge point, otherwise $O_{11}$ is the point satisfying the condition. Similarly, $O_{11}$ belongs to one field $\bar{O}_{11}$. So $O_{11}A$ is only triangular line that $O_{11}$ connects with edge point. So $O_{11}$ connects an inner triangular point $O_{12}$ that connects each edge point, as shown in Fig.2. In addition, $O_{12}$ belongs to a field $\bar{O}_{12}$ and $O_{12}A$ is the triangular line that connects $O_{12}$ with edge point, as shown in Fig.2.

$$O_{11}, A, O_{12} \text{ compose a triangle } \triangle O_{11} O_{12} A. \text{ So } \angle A O_{11} O_{12} < 180^\circ. \text{ In the same way } O_{12} \text{ connects with an inner triangular point } O_{13} \text{ that connects edge point, and } O_{12} A \text{ is only one triangular line. So } \angle A O_{12} O_{13} < 180^\circ. \text{ Thus, it can be seen that all triangular points that connect with point } O_{11} \text{ expand from } O_{11} O_{11} \text{ side.}$$

The reason that homologous points expand from $O_{1} O_{21}$ side is the same. So the number of points $V$ is infinite, or there is a triangular point that connect with two edge points, one of which comes from $O_{1z}$ and $O_{2n}$.

1 The Topological Property of Triangulation

There are two cases of triangulation which consists of two inner triangular points, as shown in Fig.3. 1° $O_{1} O_{2}$ is an inner triangular line. 2° $O_{1} O_{2}$ is not an inner triangular line.

**Definition 2**  If any two inner triangular points can be connected by some inner triangular line, then this triangulation is called compact triangulation.

Obviously, Fig.3(a) is compact triangulation, Fig.3(b) is not compact triangulation. We will discuss compact triangulation, because the result is not difficult to improve on the uncompact triangulation.