DETERMINATION OF THE BOUNDARY OF THE PLASTIC ZONE IN A MINERAL VEIN WEAKENED BY A THREE-DIMENSIONAL HOLE

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We propose a method of computing the boundary of the plastic zone formed in a neighborhood of the hole during the mining of a mineral. The problem is studied in a three-dimensional formulation. The boundary of the plastic zone is determined from the condition of continuity of the vertical normal stresses acting on the surface of contact of an elastic half-space and an elastoplastic layer. The computation is carried out for a hole having a parallelepipedal shape. One figure. Bibliography: 2 titles.

The significant concentration of stresses near the surface of a hole more than three hundred meters in depth leads to the formation of a plastic zone in the immediate neighborhood. If the elastic modulus of the mineral (coal, for example) is much less than that of the matrix, the plastic zone concentrates in the region of the coal vein adjacent to the exposed surface of the hole, while the matrix remains elastic near the boundary of the hole.

We consider a bedrock modeled by an elastic half-space with elastic modulus $E$ and Poisson coefficient $\nu$. At a depth $H$ from the earth’s surface lies a coal vein of thickness $2h$. We let the $(x, y)$-coordinate plane coincide with the interface between the matrix and the coal vein. The $z$-axis is directed vertically upward. The stressed state of the undisturbed bedrock is described by the following basic stresses:

$$\sigma_x^0 = \sigma_y^0 = -\alpha \gamma(H - z); \quad \sigma_z^0 = -\gamma(H - z); \quad \tau_{xy}^0 = \tau_{xz}^0 = \tau_{yz}^0 = 0,$$

where $\alpha$ is the coefficient of lateral thrust, and $\gamma$ is the average specific gravity of the bedrock. The stresses in the bedrock with a hole are represented as sums:

$$\sigma_x = \sigma_x^0 + \sigma_{xx}, \ldots; \quad \sigma_{yz} = \sigma_{yz}^0 + \tau_{yz}.$$

We assume that the hole is rather deep, so that the influence of the exposed surface in determining the additional components of the stresses can be neglected. The coal vein is elastically deformed in such a way that the vertical displacements of the points of the contact surface $w(x, y, 0)$ are proportional to the normal stress $\sigma_{zz}(x, y, 0)$ with coefficient of proportionality $\lambda_1$. The tangential stresses on the surface $z = 0$ will be assumed zero.

Suppose the thin plastic layer of thickness $2h$ is weakened by a cavity in the shape of a parallelepiped. We place the origin of a rectangular Cartesian coordinate system $x, y, z_l$ at the point of intersection of the diagonals of the rectangle in the middle surface of the layer. We direct the $z_l$-axis vertically upward.

The system of relations that describes the stress-strain state of the layer contains the equilibrium equations, physical equations, the Mises plasticity criterion, and the condition of incompressibility for the components of the displacement velocities. Assuming the tangential stresses are linear functions of the vertical coordinate $z_l$, we write the solution of the system in the following form:

$$\sigma_{11} = \sigma_{22} = p_1 - \frac{k}{\sqrt{2}} \cdot \frac{x + y}{h} + \sqrt{3} \kappa \left(1 - \frac{z_l^2}{h^2}\right)^{\frac{1}{2}};$$

$$\sigma_{33} = p_1 - \frac{k}{\sqrt{2}} \cdot \frac{x + y}{h}; \quad \sigma_{12} = 0; \quad \sigma_{31} = \frac{k}{\sqrt{2}} \cdot \frac{z_l}{h}; \quad \sigma_{32} = \frac{k}{\sqrt{2}} \cdot \frac{z_l}{h};$$

$$u_1 = u_0 + w_0 \frac{x}{2h} - w_0 \frac{y}{2h} - \frac{w_0}{\sqrt{2}} \left(1 - \frac{z_l^2}{h^2}\right)^{\frac{1}{2}};$$

$$u_2 = v_0 + w_0 \frac{x}{2h} + w_0 \frac{y}{2h} - \frac{w_0}{\sqrt{2}} \left(1 - \frac{z_l^2}{h^2}\right)^{\frac{1}{2}}; \quad u_3 = -w_0 \frac{z_l}{h},$$

where $p_1, u_0, v_0,$ and $w_0$ are arbitrary constants.

From the condition that there are no external loads on the faces of the hole perpendicular to the $xy$-plane we find $p_1 = -\sqrt{3}k\pi/2$.

We write the normal stresses in the plastic layer as

$$\sigma_{11} = \sigma_{22} = \sigma_{33} + \sqrt{3}k[1 - (z_1/h)^2]^{1/2}; \quad \sigma_{33} = -\frac{\sqrt{3}}{2}k\pi - \frac{k}{\sqrt{2}} \frac{x + y}{h}.$$  

As in the two-dimensional solution of L. Prandtl [1], the stress $\sigma_{33}$ is independent of the vertical coordinate $z_1$. Therefore we shall assume that the normal stress at the contact of the matrix with the plastic zone of the coal vein corresponds to the law

$$\sigma_{33} = -\frac{\sqrt{3}}{2}k\pi - \frac{k}{\sqrt{2}} \frac{(x + y)}{h} = a(x + y) + b.$$  

To find the distribution of stresses in the elastic half-space $z > 0$ we write the boundary conditions on the plane $z = 0$ as follows:

$$\sigma_{zz}(x, y, 0) = \gamma H, \quad (x, y) \in V_1;$$
$$\sigma_{zz}(x, y, 0) = \gamma H - a(x + y) - b, \quad (x, y) \in V_2 - V_1;$$
$$\sigma_{zz}(x, y, 0) = \kappa w(x, y, 0), \quad (x, y) \in P;$$
$$\tau_{xy}(x, y, 0) = \tau_{xz}(x, y, 0) = \tau_{yz}(x, y, 0) = 0, \quad (x, y) \in V_2 + P. \quad (3)$$

Here $V_1$ is a section of the hole in the vein; $V_2 - V_1$ is the plastic region around the horizontal section of the hole; $P$ is the portion of the plane $z = 0$ corresponding to contact of the elastic half-space with the elastic vein. The boundaries of the regions $V_1$ and $V_2$ will be denoted $\Gamma_1$ and $\Gamma_2$.

The sum $V_2 + P$ forms the plane $z = 0$.

The solution of the mixed problem for the half-space in the case when normal stresses are prescribed in the finite region $V_2$ of the plane $z = 0$ while outside that region the condition that the normal stresses $\sigma_{zz}$ are proportional to the vertical displacements $w$ holds is written in the following form [2]:

$$\sigma_{zz}(x, y, 0) = -\int \int_{V_2} G(x - \xi, y - \eta) \beta(\xi, \eta) d\xi d\eta, \quad (x, y) \in P. \quad (4)$$

The function $\beta(x, y)$ is determined from the integral equation

$$\beta(x, y) = p(x, y) + \int \int_{V_2} G(x - \xi, y - \eta) \beta(\xi, \eta) d\xi d\eta, \quad (x, y) \in V_2. \quad (5)$$

In relations (4) and (5)

$$G(x, y) = -\frac{\kappa}{2\pi} \int_0^\infty \frac{tI_0(\pi r)}{r} dt, \quad r = [(x - \xi)^2 + (y - \eta)^2]^{1/2}. \quad (6)$$

Here $I_0(\pi r)$ is the Bessel function of order zero, and $p(x, y)$ is the given distribution of the stresses $\sigma_{zz}(x, y, 0)$ in the region $V_2$. The quantities $\kappa$ and $\kappa_1$ are determined by the formulas

$$\kappa = \kappa_1(\lambda + 2\mu)/(2\mu(\lambda + \mu)), \quad \kappa_1 = E_1/h,$$

where $\lambda$ and $\mu$ are the Lamé elastic constants and $E$ is the elastic modulus of the layer.

The solution (4) is valid also in the case of several holes with a finite sectional area.