AN UNDERWATER ELECTRIC SURGE IN A PERFORATED PIPE

G. A. Atanov and A. N. Chuprin

To compute the current in a perforated pipe during an underwater electric surge we use the model of a porous pipe. The computations are carried out numerically in the one-dimensional formulation taking account of the nonisoentropic nature of the process. The results of test computations are analyzed. One figure. Bibliography: 7 titles.

An underwater electric surge is frequently applied in various technological processes, including cleansing the perforation of a water intake or oil well. In this case the process can be modeled as a fluid flow in a pipe with porous walls. If the divergence of fluid through the walls \( \varphi(x, t) \) is significantly less than the flux through a cross-section of the pipe, such a motion can be described in a one-dimensional formulation [1]. The outflow through the pores leads to a nonisoentropic process (the system becomes open), and the fluid parameters are no longer related by the Tait equation but by the following relation [2]:

\[
p + B = \Phi(s) = \Phi_0 \exp \left( \frac{1 - n}{F} \int_0^t u^2 \varphi \, dt \right).
\]

Here \( p \) is the pressure, \( \rho \) is the density of the fluid, \( n \) and \( B \) are constants determined by the physical properties of the fluid (for water, in particular, \( n = 7.15 \) and \( B = 300 \text{MPa} \)), \( \Phi \) is the entropic function, \( s \) is entropy, \( u \) is the current velocity in the pipe averaged over a cross section, and \( F \) is the area of a cross-section of the pipe. The subscript 0 corresponds to the state when \( p = 0 \).

The equations of motion can be conveniently written in the variables \( u, \Phi, \) and \( c \), where \( c \) is the speed of sound. Using the formula (1) we obtain the following relations connecting the parameters:

\[
c = \sqrt{\frac{n(p + B)}{\rho}}; \quad \rho = \left( \frac{c^2}{n\Phi} \right)^{1/(n-1)}; \quad p = \Phi^{1/(1-n)} \left( \frac{c^2}{n} \right)^{n/(n-1)} - B.
\]

We now introduce the dimensionless variables \( \bar{u} = u/c_0, \bar{c} = c/c_0, \bar{\varphi} = \varphi/(2R), \bar{\Phi} = \Phi/\Phi_0, \) and \( \bar{t} = t c_0 / (2R) \), where \( R \) is the radius of the pipe. (For simplicity we shall omit the bar from now on.) We shall also assume that the outflow is into a medium with counterpressure \( p_H \). Introducing the coefficient of porosity \( \mu \), defined as the ratio of the area of the pores (the orifices through which the outflow takes place) per unit length of pipe to the area of its lateral surface, we obtain the following system of dimensionless equations:

\[
\begin{align*}
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + \frac{n - 1}{2} \frac{c \partial u}{\partial x} - c \left( \frac{\partial \ln \Phi}{\partial t} + u \frac{\partial \ln \Phi}{\partial x} \right) &= -2c\mu \left( \frac{\Phi}{c^2} \right)^{1/(n-1)} \sqrt{2(n-1)(c^2 - c_H^2)}; \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{2c^2}{n - 1} \frac{\partial c}{\partial x} - \frac{n(c - 1) \partial \Phi}{\partial t} &= 4u\mu \left( \frac{\Phi}{c^2} \right)^{1/(n-1)} \sqrt{\frac{2}{n - 1}(c^2 - c_H^2)}; \quad (2) \\
\frac{\partial \ln \Phi}{\partial t} + u \frac{\partial \ln \Phi}{\partial x} &= -4numer^2 \left( \frac{\Phi}{c^2n} \right)^{1/(n-1)} \sqrt{2(n-1)(c^2 - c_H^2)}.
\end{align*}
\]

The boundary conditions are posed on the surface of the discharge channel and consist of the condition that the pressure and velocity of the medium be continuous on that surface. In computing the parameters in the discharge channel it is usually assumed that the medium in the channel is an ideal gas with effective adiabatic index \( k = 1.256 \) (cf. [3]), while the process itself is quasi-stationary. The basic relation here is the

energy balance. Denoting by $p_k$, $V$, and $x_k$ the pressure in the channel, the volume of the channel, and the surface coordinate on the channel, we obtain

$$p_k(t) = \Phi^{1/(1-n)}(x_k,t) \left[ \frac{c^2(x_k,t)}{n} \right]^{h/(n-1)} - B; \quad u(x_k,t) = \frac{dx_k}{dt}; \quad \frac{dV}{dt} + \frac{V}{k-1} \frac{dp_k}{dt} = N. \quad (3)$$

Here $N = N(t)$ is the electrical power delivered in the discharging process (usually $N$ is given experimentally [4]).

A system (2) of hyperbolic type has three families of characteristics:

$$\frac{dx}{dt} = u + c, \quad \frac{dx}{dt} = u - c, \quad \frac{dx}{dt} = u,$$

on which the following respective conditions hold (the consistency conditions):

$$\frac{du}{dt} + \frac{2}{n-1} \frac{dc}{dt} - \frac{c}{n(n-1)} \frac{d\ln \Phi}{dt} = -4\mu \left( \frac{\Phi}{c^2} \right)^{1/(n-1)} \left( \frac{n-1}{c} u^2 + c - u \right) \sqrt{\frac{2}{n-1} (c^2 - c_H^2)} = f^1;$$

$$\frac{du}{dt} - \frac{2}{n-1} \frac{dc}{dt} + \frac{c}{n(n-1)} \frac{d\ln \Phi}{dt} = 4\mu \left( \frac{\Phi}{c^2} \right)^{1/(n-1)} \left( \frac{n-1}{c} u^2 + c + u \right) \sqrt{\frac{2}{n-1} (c^2 - c_H^2)} = f^2;$$

$$\frac{d\ln \Phi}{dt} = -4\mu u^2 n \left( \frac{\Phi}{c^2} \right)^{1/(n-1)} \sqrt{2/(n-1) (c^2 - c_H^2)} = f^3.$$

The last condition coincides with the third condition of the system (2).

To solve the problem we apply a method of grid-characteristic type. It consists of integrating the consistency conditions written in terms of the partial derivatives [5] rather than the original system of equations. Since each of these conditions is a transport equation, it can be approximated by either the "left endpoint" or the "right endpoint" scheme depending on the part of the grid from which the corresponding characteristic arrives at the point where the computation is performed. Such a method is developed in [6] for movable grids.

The approach just described is a little-known interpretation of a scheme of S. K. Godunov constructed by him in order to obtain discontinuous solutions. Another widely known interpretation of this method is the "discontinuity decay" method.

The total derivatives in the directions of the characteristics of families I, II, and III and in the direction of motion of a node of the grid are respectively

$$\frac{d}{dt} \mid_1 = \frac{\partial}{\partial t} + (u + c) \frac{\partial}{\partial x}; \quad \frac{d}{dt} \mid_II = \frac{\partial}{\partial t} + (u - c) \frac{\partial}{\partial x}; \quad \frac{d}{dt} \mid_III = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x}; \quad \frac{d}{dt} \mid_c = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x},$$

where $U$ is the speed of the node. Introducing the notation $I^\pm = u \pm \frac{2}{n-1} c$, so that $u = \frac{1}{2} (I^+ + I^-)$, $c = \frac{2}{n-1} (I^+ - I^-)$, we obtain the difference formulas

$$\ln \Phi^i = \ln \Phi_i + \tau I^i \frac{1}{2} \left( U_{j-1/2} - U_i \right) \left( \frac{c_{j-1/2}}{n(n-1)} \ln \Phi_{j-1} - \ln \Phi_i \right);$$

$$I^{+i} = \frac{c_{i+1/2}}{n(n-1)} \left( \frac{c_{i+1/2}}{n(n-1)} \ln \Phi_i - \ln \Phi_i \right) + \frac{\tau}{x_k - x_{k-1}} \left[ (u + c)_{k-1} - U_i \right] \left[ \frac{c_{k-1/2}}{n(n-1)} \ln \Phi_k - \ln \Phi_{k-1} - I^+_k - I^-_k \right];$$

$$I^{-i} = \frac{c_{i+1/2}}{n(n-1)} \left( \frac{c_{i+1/2}}{n(n-1)} \ln \Phi_i - \ln \Phi_i \right) + \frac{\tau}{x_m - x_{m-1}} \left[ (u - c)_{m+1/2} - U_i \right] \left[ \frac{c_{m+1/2}}{n(n-1)} \ln \Phi_m - \ln \Phi_m + I^+_m - I^-_m \right].$$

Here the superscripts correspond to the parameters at the nodes of the grid for the time $t + \tau$ (where $\tau$ is the step size in time), and the subscripts correspond to the time $t$. The fractional index means that the