On the Isotropy of Continuized Dislocated Crystals. I. The Isotropic Lattice Distortion

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Received February 16, 1996

The geometrical theory of continuous distributions of dislocations traditionally neglects the dependence of a distribution of dislocations on the existence of point defects created by this distribution (e.g., due to intersections of dislocation lines). In this paper the influence of such point defects on metric properties of the continuized dislocated Bravais crystalline structure is assumed to be isotropic. The influence of the point defects on the distribution of dislocations is then modeled by treating dislocations as those located in a conformally flat space. This approach leads (among others) to new results concerning the geometry of glide surfaces.

1. INTRODUCTION

The influence of many dislocations on mechanical properties of a crystalline solid is described in mechanics of continua by means of the so-called geometrical theory of dislocations (e.g., Bilby et al., 1958; Kröner, 1984; Trzósowski, 1993, 1994). According to this theory, though the existence of many dislocations breaks the long-range order of a crystalline solid, nevertheless its short-range order is remarkably preserved and the dislocated crystalline solid can be locally approximately described as a (macroscopically small) part of an ideal crystal. On the other hand, it is known that the occurrence of many dislocations in a crystalline solid is accompanied by the appearance of point defects, due, e.g., to intersections of the dislocation lines. For example, two intersecting right (or left) screw dislocations produce a line of (self-) interstitials, and if one screw is right and the other left a line of vacancies is formed (Frank and Steeds, 1975). Point defects can appear also at crossover points of edge dislocations or when two parallel dislocation lines join together (Oding, 1961).

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However, it is known also that dislocations have no influence on the local metric properties of the crystal structure of the body (Kröner, 1985). Consequently, the short-range order of a dislocated Bravais crystal (with the above-mentioned secondary appearance of point defects) can be described, in a continuous limit defining the so-called continuized crystal (Kröner, 1984, 1986; Trz̆sowski, 1993), by means of a triple \((\Phi, G, g)\) (Trz̆sowski, 1994), where \(\Phi = (E_a; a = 1, 2, 3)\) is a moving (vectorial) frame globally defined on the body (identified with an open connected subset \(\mathcal{B}\) of the Euclidean point space \(E^3\)). \(G \subset SO(3)\) is a group of rotations describing material symmetries of a macroscopically homogeneous crystalline solid, and \(g, [g] = \text{cm}^2\), is a metric tensor with respect to which \(\Phi\) is orthonormal:

\[
g(X) = \delta_{ab} E^a(X) \otimes E^b(X)
\]

where \(X = (X^A)\) is a Lagrange coordinate system on the body \(\mathcal{B}\); we will use the so-called geometric frame references, i.e., dimensional coordinate systems such that \([X^A] = [dX^A] = \text{cm}, [\partial_A = \partial/\partial X^A] = \text{cm}^{-1}\) (in the cgs units system). \(\Phi^* = (E^a)\) is the moving coframe dual to \(\Phi\):

\[
E_a(X) = e^a(X) \partial_A, \quad E^a(X) = e_a(X) dX^A
\]

\[
e^a(X) e_A(X) = \delta^a_b; \quad [E_a] = \text{cm}^{-1}, \quad [E^a] = \text{cm}
\]

The moving frame \(\Phi\) is (in general) the anholonomic one:

\[
[E_a, E_b] = C_{ab}^c E_c
\]

where \([u, v] = u \circ v - v \circ u\) denotes the commutator product (bracket) of vector fields \(u\) and \(v\) considered as first-order differential operators, and smooth scalars \(C_{bc}^a\) constitute the so-called object of anholonomity (of \(\Phi\)).

The vector fields \(E_a\) define, at each point of the body (in like manner as in the case of a discrete Bravais crystalline structure), a triple of local crystallographic directions and scales of a locally Euclidean internal length measurement along them. It ought to be stressed that the base vectors \(E_a\) do not describe translational symmetries of an ideal local lattice (even in the case of a monocrystalline solid). This is because in a continuized crystal translational symmetries are lost and only rotational symmetries (of the considered crystalline material) are preserved (Trz̆sowski, 1993). The metric tensor \(g\) defining the (non-Euclidean) internal length measurement in the body represents the property of the dislocated crystalline solid that dislocations as well as the secondary point defects created by them have no influence on its local metric properties. The subgroup \(G\) [of the group \(SO(3)\) of all proper orthogonal matrices \(3 \times 3\)] can be identified with the group of point symme-