STABILITY AND STRICT POSITIVE REALNESS OF CONVEX POLYTOPES OF INTERVAL POLYNOMIALS*

WANG Zhi-zhen (王志珍)¹, WANG Long (王 龙)¹, YU Wen-sheng (邵文生)²

(1. Department of Mechanics and Engineering Sciences, Peking University, Beijing 100871, P R China;
2. Institute of Automation, Chinese Academy of Sciences, Beijing 100080, P R China)

(Communicated by LIN Zong-chi)

Abstract: For an uncertain system described by convex combination of interval polynomials, its Hurwitz-stability can be guaranteed by certain subset composed of vertices and edges. Furthermore, the testing set does not increase when the order of given system increases.

Key words: interval polynomials; convex combination of polynomials; Hurwitz-stability; value set

CLC number: O175 Document code: A

Introduction

Among the researches on parametric uncertainty systems, Kharitonov [1] gave the first seminal result, i.e., four vertices criterion, where a family of real interval polynomials with constant degree are considered. When the coefficients are complex, the former result was extended to eight vertices. Motivated by such work, the study on robustness stability under parameter uncertainties attract considerable interest. Box Theorem [2,3] adapts to the systems of linear combinations of interval polynomials, which reduced the testing set to a subset independent to the order of systems. In 1988, A. C. Bartlett, C. V. Hollot and L. Huang obtained the known Edge Theorem [4]. It is an equivalent condition to stability of polytopes of polynomials with regard to simple connected region. But since the result is related to the order of system, the testing set increases in exponential form when the number of uncertain parameters increases. In fact, for convex combination of interval polynomials, Kharitonov-like condition holds. Based on former work, using analysis method, this paper gives an efficient equivalent condition on the robust stability of systems under parametric uncertainties.

First, we give some definitions and standard notations.

Definition 1 Given interval polynomial $f(s) = \sum_{i=0}^{n} f_is^i$, $f_i \in [f_i^-, f_i^+]$, construct $R_f$, *Received date: 2000-06-30; Revised date: 2001-09-18

Foundation item: the National Natural Science Foundation of China (69925307);

Biography: WANG Zhi-zhen (1974 – ), Doctor
$R_f, I_f, \overline{I}_f$ as below
\[
R_f = f_0 + f_2 s^2 + f_4 s^4 + \cdots, \\
\overline{R}_f = \overline{f_0} + \overline{f_2} s^2 + \overline{f_4} s^4 + \cdots, \\
I_f = f_1 + f_3 s^2 + f_5 s^4 + \cdots, \\
\overline{I}_f = \overline{f_1} + \overline{f_3} s^2 + \overline{f_5} s^4 + \cdots.
\]

**Definition 2** Interval polynomials $f_1, \cdots, f_m$ are in consistency, if there exist two real numbers $\mu, \nu \in [0, 1]$ such that
\[
f_i(s) = [\mu R_f + (1 - \mu)\overline{R}_f] + [\nu I_f + (1 - \nu)\overline{I}_f], \quad i = 1, \cdots, m.
\]

**Definition 3** Value set $\mathcal{V}(j\omega) = \bigcup_{f \in \mathcal{V}} \{f(j\omega)\}$, $\omega \in \mathbb{R}$.

**Definition 4** $\mathcal{V}$ is Hurwitz-stable, if $\forall f \in \mathcal{V}$, all roots of polynomial $f(s)$ belong to the open left half complex plane.

**Definition 5** Given a family of interval polynomials $f(s) = \sum_{i=1}^{m} f_i s^i$, $f_i \in [f_i, \overline{f}_i]$, then $K^0_f = \{f^1_k, f^2_k, f^3_k, f^4_k\}$ is called the Kharitonov vertex set of $f(s)$; $E^0_f = \{\lambda f^0_k + (1 - \lambda)f^0_k', (s, t) \in \{(1,2), (2,4), (4,3), (3,1)\}, \lambda \in [0, 1]\}$ is called the Kharitonov exposed edge set of $f(s)$, where
\[
f^1_k = f_0 + f_1 s + f_2 s^2 + f_3 s^3 + \cdots, \\
f^2_k = \overline{f_0} + \overline{f_1} s + \overline{f_2} s^2 + \overline{f_3} s^3 + \cdots, \\
f^3_k = \overline{f_0} + \overline{f_1} s + \overline{f_2} s^2 + \overline{f_3} s^3 + \cdots, \\
f^4_k = \overline{f_0} + \overline{f_1} s + \overline{f_2} s^2 + \overline{f_3} s^3 + \cdots.
\]

It is easy to see that
\[
f^1_k = R_f + sI_f, \quad f^2_k = \overline{R}_f + s\overline{I}_f, \\
f^3_k = \overline{R}_f + s\overline{I}_f, \quad f^4_k = R_f + sI_f.
\]

1 **Stability**

Consider the convex combination of $m$ interval polynomials, that is
\[
\mathcal{V} = \left\{ \sum_{i=1}^{m} \lambda_i f_i ; \sum_{i=1}^{m} \lambda_i = 1, \quad f_i \text{ is an interval polynomial of Hurwitz-stable} \right\} \quad (1)
\]
Denote $f_i$ as follows:
\[
f_i = \sum_{k=0}^{s} f_{i}^{k} s^k, \quad f_{i}^{k} \in [f_i^k, \overline{f}_i^k],
\]
where $k$ denotes the subscript.

**Theorem 1** $\forall \omega \in \mathbb{R}$, $\mathcal{V}(j\omega) = \left\{ (\sum_{i=1}^{m} \lambda_i f_i)(j\omega) ; f_1, \cdots, f_m \text{ are in consistent} \right\}$.

**Proof** For all $g \in \mathcal{V}$, there exist $\lambda_1, \cdots, \lambda_m \in [0, 1]$, $\sum_{i=1}^{m} \lambda_i = 1$ such that $g = \sum_{i=1}^{m} \lambda_i f_i$. If it equals to $\sum_{i=1}^{m} \mu_i p_i$, where $\mu_i \in [0, 1]$, $\sum_{i=1}^{m} \mu_i = 1$ and $p_1, \cdots, p_m$ are in