COMPUTING THE EIGENVECTORS OF A MATRIX WITH MULTIPLEX EIGENVALUES BY SVD METHOD *

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Abstract: Every matrix is similar to a matrix in Jordan canonical form, which has very important sense in the theory of linear algebra and its engineering application. For a matrix with multiplex eigenvalues, an algorithm based on the singular value decomposition (SVD) for computing its eigenvectors and Jordan canonical form was proposed. Numerical simulation shows that this algorithm has good effect in computing the eigenvectors and its Jordan canonical form of a matrix with multiplex eigenvalues. It is superior to MATLAB and MATHEMATICA.

Key words: multiplex eigenvalue; eigenvector; eigenvector chain; Jordan canonical form

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Introduction

In the theory of linear algebra and its engineering application, more and more attention is paid to computing the eigenvectors and Jordan canonical form of a matrix. For the mode analysis of vibration system and the design of control system, we also need to consider how to compute the eigenvectors of a matrix with multiplex eigenvalues. In the case of a matrix without multiplex eigenvalues, there have been several methods to compute the eigenvalues and corresponding eigenvectors, further, we can obtain its Jordan canonical form. Currently, many commercial softwares which are widely used have been developed by these methods, such as EISPACK, MATLAB[1] and MATHEMATICA[2], etc. In the latest version of MATLAB, the case of a matrix with multiplex eigenvalues is considered too. However, when the order is more than 5 and there are multiplex eigenvalues, the results computed with MATLAB or MATHEMATICA tool are often dissatisfying. They are only the approximate solutions with higher error. The main cause leading to the above results lies in that these softwares adopt the inverse power method[3] to

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compute the eigenvectors. In principle, it is not suitable to compute a matrix with multiplex eigenvalues. While we use the symbolic operation to compute a matrix, its elements must be the integers or ratios of small integers. It is not fit for the engineering problem.

In Ref. [4], we proposed an algorithm to compute the multiplex eigenvalues of a matrix and got the satisfying results. Now we use singular value decomposition method to compute the eigenvectors and Jordan canonical form of a matrix. The main idea of the algorithm is as follows. Firstly, compute the multiplex eigenvalues of a matrix. Secondly, determine the eigenvectors belonging to these eigenvalues and the number of corresponding Jordan blocks by singular value decomposition method. Thirdly, according to the special form of the Jordan block, determine the eigenvector chain corresponding to each Jordan block. Finally, compute the eigenvectors and Jordan canonical form of the matrix.

Numerical examples show that this algorithm has good effect in computing the eigenvectors and its Jordan canonical form of a matrix with multiplex eigenvalues.

1 Basic Principle of the Algorithm

By the theory of linear algebra, for any n-square matrix $A$, if $A$ has $n$ distinct eigenvalues, then $A$ is similar to a diagonal matrix, i.e. there exists an invertible matrix $W$ so that $W^{-1}AW$ is diagonal, whose main diagonal entries are the eigenvalues of $A$. However, if the number of distinct eigenvalues of $A$ is less than $n$, i.e. the characteristic polynomial of $A$ has multiplex roots, then whether a matrix can be diagonalized will be determined by the dimensions of eigenspaces corresponding to the multiplex eigenvalues. While the sum of the dimensions of eigenspaces equals to the order of square matrix $A$, $A$ is still similar to a diagonal matrix, whose main diagonal entries are determinate except their sequence. These entries are the whole eigenvalues of matrix $A$.

While the sum of the dimensions of eigenspaces is less than the order of $A$, $A$ is not diagonalizable. But we can still find an invertible matrix $W$ so that $W^{-1}AW$ has the following form:

$$J = \begin{bmatrix} J_1 & 0 \\ J_2 & \ddots \\ 0 & \ddots & J_t \end{bmatrix},$$

where

$$J_i = \begin{bmatrix} J_{i1} & 0 \\ J_{i2} & \ddots \\ 0 & \ddots & J_{in_i} \end{bmatrix}, \quad J_{ij} = \begin{bmatrix} s_i & 1 & 0 \\ s_i & 1 & \ddots \\ 0 & \ddots & s_i \end{bmatrix},$$

$t$ is the number of distinct eigenvalues of $A$. $n_i$ is the dimension of eigenspace corresponding to the eigenvalue $s_i$. Square matrix $J_{ij}$ ($i = 1, 2, \cdots, t; j = 1, 2, \cdots, n_i$) is called a Jordan block corresponding to the eigenvalue $s_i$, whose order is $n_i$ ($\sum_{j=1}^{n_i} n_{ij}$ is the multiplex numbers of the eigenvalue $s_i$). The matrix $J$ appearing with the above form is called the Jordan canonical form.