RESEARCH ON THE COMPANION SOLUTION FOR A THIN PLATE IN THE MESHLESS LOCAL BOUNDARY INTEGRAL EQUATION METHOD *

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Abstract: The meshless local boundary integral equation method is a currently developed numerical method, which combines the advantageous features of Galerkin finite element method (GFEM), boundary element method (BEM) and element free Galerkin method (EFGM), and is a truly meshless method possessing wide prospects in engineering applications.

The companion solution and all the other formulas required in the meshless local boundary integral equation for a thin plate were presented, in order to make this method apply to solve the thin plate problem.

Key words: thin plate; companion solution; meshless local boundary integral equation method

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Introduction

The meshless local boundary integral equation method presented by Atluri et al. [1] was firstly applied to solve potential problems described by a Poisson equation. The two-dimensional elasticity problem was recently considered by Long and Xu [2] by the local boundary integral equation (LBIE) method.

LBIE method is a truly meshless method, which means that the discretization is independent of geometric subdivision into elements or cells, and is only based on a set nodes (ordered or scattered) over a domain in question. The method combines the advantageous features of all the three methods: Galerkin Finite Element Method (GFEM), Boundary Element Method (BEM) and Element Free Galerkin Method (EFGM). Results obtained by Atluri et al. [1] for the 2-D potential problem and by Long and Xu [2] for the 2-D elasticity problem show that the method

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possesses an excellent rate of convergence, a good stability and a high accuracy.

When a boundary value problem defined by most differential operators is solved by LBIE method, besides the need of the fundamental solution in an infinite space in question, the companion solution over a regular domain (a sphere for 3-D problem, and a circle for 2-D problem) in question is also required. Companion solutions for 2-D potential and elasticity problems were given by Atluri et al.\textsuperscript{11} In the present work, the companion solution for a thin plate problem is analytically solved, then all the formulas needed for solving the thin plate problem by LBIE method are formulated, to apply LBIE method to solve the thin plate problem.

1 Local Boundary Integral Equation for a Thin Plate

The global boundary integral equation for a Kirchhoff elastic plate is as follows\textsuperscript{[3]}:

\[
C(\xi) w_k(\xi) = \int_\Gamma \left[ w_k^*(\xi, x) V_n(x) - \theta_{nk}(\xi, x) M_n(x) - V_{nk}(\xi, x) w(x) + M_{nk}(\xi, x) \theta_n(x) \right] d\Gamma + \sum_T \left\{ w_k^*(\xi, x)[M_i(x)]^+ - w(x)[M_{nk}^*(\xi, x)]^+ \right\} + \int_T w_k^*(\xi, x) q(x) d\Omega \quad (k = 1, 2),
\]

where \(\xi\) is a source point, and \(x\) is a field point,

\[
w_2(\xi) = \frac{\partial w_1(\xi)}{\partial n(\xi)}, \quad w_2^*(\xi, x) = \frac{\partial w_1^*(\xi, x)}{\partial n(\xi)},
\]

\(C(\xi)\) is a constant coefficient depending upon the shape of the global boundary, which is

\[
C(\xi) = \begin{cases} 
1, & \xi \in \Omega, \\
1/2, & \xi \in \Gamma \text{ and } \xi \text{ is a node on a smooth boundary}, \\
\gamma/2\pi, & \xi \in \Gamma \text{ and } \xi \text{ is a node on boundary corners}, \\
0, & \xi \in (\Omega + \Gamma),
\end{cases}
\]

where \(\gamma\) is the internal angle at a corner (see Fig. 1). The meanings of other symbols in Eq. (1) are as follows:

\[
\begin{align*}
\theta_n(x) &= \frac{\partial w(x)}{\partial n(x)}, \\
M_n(x) &= -\frac{D}{2}(1 - \mu) \left[ \frac{1 + \mu}{1 - \mu} \nabla^2 w + \cos 2\beta L_1(w) + 2\sin 2\beta L_2(w) \right], \\
M_i(x) &= D(1 - \mu) \left[ \frac{1}{2} \sin 2\beta L_1(w) - \cos 2\beta L_2(w) \right], \\
V_n(x) &= Q_n(x) + \frac{\partial M_i}{\partial s} = -D \frac{\partial}{\partial n(x)}(\nabla^2 w) + L_3(M_i),
\end{align*}
\]

where

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{r^2 \partial \theta^2},
\]