A SEMI-ANALYSIS METHOD OF DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS UNDER COMPLICATED BOUNDARY CONDITIONS *

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Abstract: Based on a method of finite element model and combined with matrix theory, a method for solving differential equation with variable coefficients is proposed. With the method, it is easy to deal with the differential equations with variable coefficients. On most occasions and due to the nonuniformity nature, nonlinearity property can cause the equations of the kinds. Using the model, the satisfactory valuable results with only a few units can be obtained.

Key words: differential equation with variable coefficients; equivalent parameter; solution in the domain; solution of semi-analysis

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Introduction

In the most cases, as far as the solution to the differential equation with variable coefficients is concerned, there has been no any satisfactory answer. For this reason, the scholars both at home and abroad have done some research just as indicated in Refs. [1], [2] and [3], etc. The authors in this paper have used the finite element model and the metric function theory in combination with adjustable parameter model with variable coefficients to simplify the solution to the differential equation with variable coefficients into the feature value as well as the feature vector function problem. Also, the authors advanced a kind of semi-analysis method in the whole field, on the basis of which, most of the differential equations with variable coefficients and the differential equation group with variable coefficients can be solved.

1 Models of Equivalent Parameter

In dynamic problem, the controlling equation that is often met with is a problem containing

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a broad feature value of unknown parameters and feature vector. It is just here that a differential equation with variable coefficient containing an unknown parameter \( k \) and complex homogeneous boundary conditions can be expressed in the following:

\[
\sum_{i=1}^{m} p_i(x) s^{(i)}(x) + kg'(x) s(x) = 0, \quad (1)
\]

\[
\sum_{k=1}^{j} a_k(x) s^{(j-k)}(x) l_{x=a} = 0, \quad \sum_{k=1}^{j} b_k(x) s^{(j-k)}(x) l_{x=b} = 0, \quad (2)
\]

where \( p_i(x) \) \((i = 1, 2, 3, \cdots)\) and \( g(x) \) are the variable coefficients, \( s^{(m)}(x) \) is the highest order derivative for unknown function \( s(x) \), \( k \) is the unknown parameter, it is about problem of characteristic value; \( a, b \) are the starting point and end point of problem, the \( a_k, b_k \) \((k = 1, 2, \cdots)\) are the known functions about \( x \), in general \( j < m \).

Using finite element model, we discrete Eq. (1) and introduce non-dimensional coordinate:

\[
\xi_i = \frac{x - x_{i-1}}{x_i - x_{i-1}} = \frac{x - x_{i-1}}{l_i}, \quad (\xi_i \in [0, 1]). \quad (3)
\]

Here \( l_i \) is the length of \( i \)th unit \((i = 1, 2, 3, \cdots, n)\). In \( i \)th sub-domain governing equation (1) can be written as

\[
p_{1i}(\xi_i) s_{1}^{(1)}(\xi_i) l_i^{m-1} + p_{2i}(\xi_i) s_{1}^{(2)}(\xi_i) l_i^{m-2} + \cdots + p_{mi}(\xi_i) s_{1}^{(m)}(\xi_i) + g(\xi_i) l_i^{m-1} s_i(\xi_i) = 0. \quad (4)
\]

In Eq. (4), selecting very important function about functions \( p_{1i}(\xi), p_{2i}(\xi), \cdots, g(\xi) \), there is much more influence on governing factions for differential equations, in general, recommendable is more rate of change. According to integration middle value theorem, from governing function calculate equivalent parameter \( \xi_i^* \), shown as follows:

\[
\xi_i^* = \int_0^1 \frac{\xi p(\xi l_i + x_{i-1}) d\xi}{\int_0^1 p(\xi l_i + x_{i-1}) d\xi}, \quad (5)
\]

In Eq. (5), \( p(\xi l_i + x_{i-1}) \) is selecting governing function. According to Eq. (5), equivalent parameter can be got (it is equivalent parameter about \( i \) sub-domain). Obviously, equivalent parameter changes with sub-domain. Eq. (4) may be written as

\[
\sum_{j=1}^{m} p_{ji}(\xi_i^* l_i + x_{i-1}) s^{(j)}(\xi) + l_i^{m-1} k_{ji}(\xi_i^* l_i + x_{i-1}) s_i(\xi) = 0. \quad (6)
\]

2 Problems of Generalized Characteristic Function and Characteristic Vector

Equation (6) can be changed into differential equation systems, written as

\[
\frac{d}{d\xi} \{\delta_i(\xi)\} = [A_i] \{\delta_i(\xi)\}, \quad (7)
\]

where \( \{\delta_i(\xi)\} = \left\{ s_i(\xi), \frac{ds_i(\xi)}{d\xi}, \frac{d^2s_i(\xi)}{d\xi^2}, \cdots, \frac{d^{m-1}s_i(\xi)}{d\xi^{m-1}} \right\}^T \).

Suppose Eq. (7) matrix \([A_i]\) with \( m \) characteristic values \( \lambda_i \) and characteristic vectors of linear independence \( \{x_i\} \). Marking sub-domain of characteristic values and characteristic vectors can be denoted respectively (The general case is complex vector):