SENSITIVITY COEFFICIENTS OF SINGLE-PHASE FLOW IN LOW-PERMEABILITY HETEROGENEOUS RESERVOIRS *

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Abstract: Theoretical equations for computing sensitivity coefficients of wellbore pressures to estimate the reservoir parameters in low-permeability reservoirs conditioning to non-Darcy flow data at low velocity were obtained. It is shown by a lot of numerical calculations that the wellbore pressures are much more sensitive to permeability very near the well than to permeability a few gridblocks away from the well. When an initial pressure gradient exists sensitivity coefficients in the region are closer to the active well than to the observation well. Sensitivity coefficients of observation well at the line between the active well and the observation well are influenced greatly by the initial pressure gradient.

Key words: non-Darcy flow through porous media; permeability; porosity; sensitivity coefficient; inverse problem; low-permeability reservoir

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Introduction

Sensitivity coefficients express the change rate of observation data on model parameters, and represent sensitivity degree of estimated parameters on model. It is important to calculate them in reservoir descriptions.

Jacquard and Jian¹ presented the first procedure of numerically computing sensitivity coefficients for estimation of permeability in a two-dimensional reservoir from pressure data based on an electric circuit analogue. Anterion et al.² introduced the gradient simulator method to petroleum engineering, by solving a linear system obtained by differentiating the matrix form of the finite-difference equations with respect to a model parameter, e.g., a gridblock value of permeability or porosity. The advantage of the gradient simulator method is that it reduces to

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solving a matrix problem with multiple right-hand side vectors. The difficulty is that if we wish to estimate permeabilities and porosities at several thousand gridblocks, then we have several thousand right-hand sides. Although Killough et al. developed a faster iterative solve for this problem, the effort required for each sensitivity is still of the order of 10% for this problem.

In the gradient simulator method, one actually obtains the sensitivity of all gridblock pressures to each model parameter. For the most part, this is useless information because one typically only has measurements at wells. In essence, it is the computation of this useless information that detracts from the computational efficiency of the gradient simulator method. To avoid this inefficiency, Chu et al. used the basic ideas of Tang et al. to develop a modified generalized pulse-spectrum technique (MGPST), to estimate the sensitivity of wellbore pressures to reservoir simulator gridblock permeabilities and porosities. The MGPST yields reasonably accurate estimate of sensitivity coefficients related to the permeability field, unfortunately it doesn’t yield accurate values of sensitivity coefficient related to the porosity field.

Carter et al. presented an elegant derivation to compute sensitivity coefficients for two-dimensional single-phase flow problems. Carter’s procedure for single-phase flow has been extended to three-dimensional problems in a computationally efficient way by He et al.. Richard reviewed parameter estimation methods including some discussions of methods for calculating sensitivity coefficients.

Applied condition of the methods above is Darcy flow in a middle or high permeability reservoir. For a low-permeability reservoir, the flow doesn’t obey Darcy rules because threshold pressure gradient at low velocity. The sensitivity coefficient, especially, at a wellbore, will be obtained in a low-permeability reservoir.

1 Derivative of Sensitivity Coefficients

If \( p_{wf}(t) \) denotes the pressure response at a well for a given reservoir, then the relevant sensitivity coefficients at any time \( t \), are

\[
\frac{\partial p_{wf}(t)}{\partial k_{x,i,m,n}} \quad \frac{\partial p_{wf}(t)}{\partial k_{y,i,m,n}} \quad \frac{\partial p_{wf}(t)}{\partial k_{z,i,m,n}}
\]

for all \((i, m, n)\). \( p \) is pressure at any point in a reservoir, MPa; \( t \) denotes time, h; \( k \) denotes formation permeability, \( \mu m^2 \); \( \phi \) denotes porosity; superscripts \( i, m, n \) denotes \( x, y, z \) direction, \( x, y, z \) is respectively length, m; \( wf \) represents wellbore.

For the three-dimensional single flow problem considered in a low-permeability reservoir, the governing flow equation can be written in oil field units as follows:

\[
\frac{c_1}{\mu} \nabla \cdot \left[ k \right] \left( 1 - \lambda H_d \right) \nabla p_d - V \frac{\partial p_d}{\partial s} = -Q, \quad (1a)
\]

\[
p_d \big|_{s=0} = 0, \quad \nabla p_d \cdot n = 0, \quad (1b, c)
\]

here \( V = \phi c_i \), \( p_d = p_i - p \), \( p_i \) is initial pressure in a reservoir, MPa; \( c_i \), total system compressibility, MPa; \( \mu \) is fluid viscosity, mPa·s; \( Q \) is flow rate, m³/h; \( \lambda \) is threshold pressure gradient, MPa/m; \( H_d(x,y,z) = 1/\lambda \nabla p_d \big| s \) is function of pressure gradient.

Consider the same problem with a small perturbation in the permeability field

\[
\frac{c_1}{\mu} \nabla \cdot \left[ k + \delta k \right] \left( 1 - \lambda H_d \right) \nabla p_d - V \frac{\partial p_d}{\partial s} = -Q, \quad (2a)
\]