AN IMPROVED ARC-LENGTH METHOD AND APPLICATION IN THE POST-BUCKLING ANALYSIS FOR COMPOSITE STRUCTURES

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Abstract: Based on the conventional arc-length method, an improved arc-length method with high-efficiency is proposed. The weighted modifications with respect to the variation of structural stiffness and extra-interpolation modification by using the information of known equilibrium points are introduced to improve the incremental arc-length. An approximate expansion method for the accumulated and expected arc-length is used to ensure the convergence at given load levels in large range of applications. Numerical results show that the improved arc-length method has well adaptability and higher efficiency in the post-buckling analysis of plates and shells structures for tracing whole load-deflection path and obtaining the convergence values at any specified load levels.

Key words: nonlinear; finite element analysis; arc-length method

Introduction

At present, the widely used arc-length methods for the finite element analysis of structures (such as the solution of post-buckling path), which are firstly proposed by Riks and Wempner at 1979 and then modified by Crisfield and Roma [1-5], are the most efficient methods that can achieve not only the loading limit of structures in the analysis and design of engineering structures but also tracing the whole loading path from the beginning, then loading little by little till to destruction.

With the development of the modern structures in the direction of the upsizing and the complication, an increasing number of requirements are brought forward to mechanics analysis by engineering design. For example, how to determine the state of deformation and the mechanics response at the predefined load levels, or how to converge at given load levels. But for the solution in this purpose, the number of load step will often increase dramatically, which follows...
to reduce the efficiency of solution. So in order to enlarge the applicable range of the arc-length method, the key problem still focuses on how to choose an appropriate loading increment to speed up the iteration process and improve the calculation efficiency. For this reason the research on improving the arc-length method is still up and doing in recent decades.

Based on the Crisfield arc-length method, two strategies are proposed in this paper to modify the arc-length: one is weighing modified arc-length according to the change of stiffness parameter; another one is external interpolating modified arc-length using the information of the known convergence points. An efficient arc-length method is proposed with these two approaches above. The bond of two modified strategies makes an improved arc-length method having strong adaptability and high efficiency, while the method can also achieve the convergence to the predefined load levels in the post-buckling analysis for plate/shell structures. Numerical examples demonstrate the apparent improvement of the efficiency for the solution, not only in tracing the whole loading path but also in converging at the predefined load levels.

1 Two Strategies for Improving Arc-length Method

1.1 General arc-length method

In the nonlinear static analysis, the incremental control equation is

$$\left[ K_T \right] \{ \Delta \bar{u} \} = \Delta \lambda \{ F_T \} + \{ R \},$$

where $\left[ K_T \right]$ is current tangent stiffness matrix; $\{ \Delta \bar{u} \}$ is the incremental nodal displacement vector; $\Delta \lambda$ is the incremental load factor; $\{ F_T \}$ is a fixed reference load vector; $\{ R \}$ is the residual nodal vector of the nodal forces.

In the solution of a system of nonlinear equations for a finite element model of a structure using the arc-length method, the accumulated displacements of the $i$-th loading step after the $j$-th iteration are given by

$$\{ \Delta \bar{u}_i \} \mid _j = \{ \Delta \bar{u}_i \} \mid _{j-1} + \Delta \lambda \{ \bar{u}_i \} + \{ \Delta \bar{u}_R \} \mid _j,$$

in which $\Delta \lambda \mid _j$ is the change of the load factor of the $j$-th iteration within the $i$-th loading step; and $\{ \bar{u}_i \} \mid _i$ is the tangent displacement vector under an arbitrary reference load level; and $\{ \Delta \bar{u}_R \} \mid _i$ is the vector of displacement increments due to residual forces.

The change of load factor $\Delta \lambda$ is determined by the $j$-th arc-length increment $l_i$, which usually satisfies the iteration equation proposed by Crisfield $^{[1]}$

$$\{ \Delta \bar{u}_a \} \mid _j \{ \Delta \bar{u}_a \} \mid _j = l_i^2.$$

And the arc-length $l_i$ is updated at each cycle according to the equation proposed by Bellini:

$$l_i = l_{i-1} (J_d / J_{i-1})^{1/2},$$

where $l_i$ is the arc-length for the $i$-th loading step; $J_d$ is the desired number of iterations for the $i$-th loading step; $J_{i-1}$ is the number of iterations required to converge in the $(i-1)$-th loading step.

1.2 Weighing modified arc-length

To improve the efficiency of calculation, it's necessary to choose step values ($\Delta \lambda$ and