PRELIMINARY ANALYSIS OF SKATING MOTION*

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Abstract: A mechanical model of skating motion was founded, and its solution was obtained by using the Routh's equations in nonholonomic dynamics. The two kinds of common, local meaning and scleronomic motions were discussed in detail. The computational results turn out in good agreement with observations.

Key words: nonholonomic dynamics; skating; scleronomic motion
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Introduction

The sleigh problem is a classic one in nonholonomic dynamics. The solutions to the problem were obtained by C. A. Li, a[1] and C. Carathodory [2]. The reference [2] presented a few questionable points in the solutions. Another solution was offered in the reference [3]. But the model of sleighs in these solutions is not suitable for skater's actual skating motion. This paper presents a mechanical model for skating motion. By use of Routh’s equations in nonholonomic dynamics, a solution is derived, and two kinds of common, local meaning and scleronomic motions are discussed in detail.

1 The Foundation of the Model

The bottom blade of an ice skate which contacts with ice plane has an approximate rectilinear segment about 150 mm long as shown in Fig. 1[4]. A skater is skating on horizontal ice plane only with one leg (Fig. 2). We consider the skater and the skate as a whole. The center of mass of the skater of mass \( m \) is situated at the point \( G \). For simplicity, the 1, 2 and 3 axes are taken as the principal axes of the system. Let \( A, B \) and \( C \) designate the moments of inertia of the skater with respect to the 1, 2 and 3 axes respectively. Assume that the point \( K \) on the 1 axis is the lowest point in the skate. Since there is an approximate rectilinear segment in the blade, the rotation of the skater about the 3 axis is not considered. Therefore, the rotation of the skater about the center of mass \( G \) is described by the angles \( \phi \) and \( \theta \), as shown in Fig. 2. The coordinates of the point \( G \) are \( (x, y, z) \), and the coordinates of the point \( K \) are \( (x_K, y_K, 0) \). We let \( a \) denote the distance between the points \( G \) and \( K \). Then,

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The equation of nonholonomic constraint imposed on the skate is
\[ x_K \sin \varphi - y_K \cos \varphi = 0, \] (2)
where overdots denote differentiation with respect to time \( t \). Differentiating Eqs. (1) and substituting them into Eq. (2), we obtain another form of the equation of constraint
\[ x \sin \varphi - y \cos \varphi - a \dot{\theta} \sin \theta = 0. \] (3)

In coordinate system G123, the angular velocity of the system is
\[ \mathbf{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} - \dot{\varphi} \sin \theta \\ \dot{\theta} \\ \dot{\phi} \cos \theta \end{bmatrix}. \] (4)

The above expression is similar to the angular velocity of the rolling disc in the rolling disc problem\(^5\), but there is no spin angle \( \psi \) in (4). We use \( T \) to designate the kinetic energy of the system:
\[ T = \frac{1}{2} m (x^2 + y^2 + z^2) + \frac{1}{2} (A \omega_1^2 + B \omega_2^2 + C \omega_3^2). \]

Since \( z = a \sin \theta \), using (4), we obtain
\[ T = \frac{1}{2} m (x^2 + y^2) + \frac{1}{2} (ma^2 \cos^2 \theta + B) \dot{\theta}^2 + \frac{1}{2} (A \sin^2 \theta + C \cos^2 \theta) \dot{\psi}^2. \] (5)

Let \( V \) be the potential energy of the system. Then
\[ V = m g a \sin \theta. \] (6)

Substituting Eqs. (5) and (6) into Routh's equations\(^1\) in nonholonomic dynamics and taking