PRELIMINARY ANALYSIS OF SKATING MOTION*

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Abstract: A mechanical model of skating motion was founded, and its solution was obtained
by using the Routh’s equations in nonholonomic dynamics. The two kinds of common, local
meaning and scleronomic motions were discussed in detail. The computational results turn out in
good agreement with observations.

Key words: nonholonomic dynamics; skating; scleronomic motion

Introduction

The sleigh problem is a classic one in nonholonomic dynamics. The solutions to the problem
were obtained by C. A. LiǎnabīrtH(1898) and C. Carathodory(1933)[1]. The reference [2]
presented a few questionable points in the solutions. Another solution was offered in the reference
[3]. But the model of sleighs in these solutions is not suitable for skater’s actual skating motion.
This paper presents a mechanical model for skating motion. By use of Routh’s equations in
nonholonomic dynamics, a solution is derived, and two kinds of common, local meaning and
scleronomic motions are discussed in detail.

1 The Foundation of the Model

The bottom blade of an ice skate which contacts with ice plane has an approximate rectilinear
segment about 150 mm long as shown in Fig. 1[4]. A skater is skating on horizontal ice plane
only with one leg (Fig. 2). We consider the skater and the skate as a whole. The center of mass
of the skater of mass m is situated at the point G. For simplicity, the 1, 2 and 3 axes are taken as
the principal axes of the system. Let A, B and C designate the moments of inertia of the skater
with respect to the 1, 2 and 3 axes respectively. Assume that the point K on the 1 axis is the
lowest point in the skate. Since there is an approximate rectilinear segment in the blade, the
rotation of the skater about the 3 axis is not considered. Therefore, the rotation of the skater about
the center of mass G is described by the angles \( \phi \) and \( \theta \), as shown in Fig. 2. The coordinates of
the point G are \((x, y, z)\), and the coordinates of the point K are \((x_K, y_K, 0)\). We let \( a \) denote
the distance between the points G and K. Then,
The equation of nonholonomic constraint imposed on the skate is
\[ x_K \sin \varphi - y_K \cos \varphi = 0, \]
where overdots denote differentiation with respect to time \( t \). Differentiating Eqs. (1) and substituting them into Eq. (2), we obtain another form of the equation of constraint
\[ x \sin \varphi - y \cos \varphi - a \dot{\varphi} \sin \theta = 0. \]
In coordinate system G123, the angular velocity of the system is
\[
\omega = \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix} = \begin{bmatrix}
- \varphi \sin \theta \\
\dot{\varphi} \\
\varphi \cos \theta
\end{bmatrix}.
\]
The above expression is similar to the angular velocity of the rolling disc in the rolling disc problem\(^5\), but there is no spin angle \( \psi \) in (4). We use \( T \) to designate the kinetic energy of the system:
\[
T = \frac{1}{2} m(x^2 + y^2 + z^2) + \frac{1}{2} (A\omega_1^2 + B\omega_2^2 + C\omega_3^2).
\]
Since \( z = a \sin \theta \), using (4), we obtain
\[
T = \frac{1}{2} m(x^2 + y^2) + \frac{1}{2} (ma^2 \cos^2 \theta + B) \dot{\varphi}^2 + \frac{1}{2} (A \sin^2 \theta + C \cos^2 \theta) \dot{\theta}^2.
\]
Let \( V \) be the potential energy of the system. Then
\[
V = mg \sin \theta.
\]