NEW PRINCIPLES OF WORK AND ENERGY AS WELL AS POWER AND ENERGY RATE FOR CONTINUUM FIELD THEORIES *

DAI Tian-min (戴天民)

(Center for the Application of Mathematics and Department of Mathematics, Liaoning University, Shenyang 110036, P R China)

(Paper from DAI Tian-min, Member of Editorial Committee, AMM)

Abstract: New principles of work and energy as well as power and energy rate with cross terms for polar and nonlocal polar continuum field theories were presented and from them all corresponding equations of motion and boundary conditions as well as complete equations of energy and energy rate with the help of generalized Piola’s theorems were naturally derived in all and without any additional requirement. Finally, some new balance laws of energy and energy rate for generalized continuum mechanics were established. The new principles of work and energy as well as power and energy rate with cross terms presented in this paper are believed to be new and they have corrected the incompleteness of all existing corresponding principles and laws without cross terms in literatures of generalized continuum field theories.

Key words: new principles of work and energy; power and energy rate; generalized Piola’s theorem; new equations of energy and energy rate; generalized continua

CLC number: O33 Document code: A

Introduction

1999 is the 90th anniversary of the publication of E. F. Cosserat’s book “Theorie des Corps Deformable”[1], which was the foundation stone and now is still a guiding monograph in the study of generalized continuum field theories. However, it was not till the publication of papers of Ericksen and Truesdell[2] and Guenther[3] in 1958 that extensive developments of various theories have taken place all over the world. Down to date the number of published papers on generalized continuum mechanics is very large. Some excellent monographs of various related theories exist, e. g., Kröener[4], Edelen[5], Stojanovic[6], Eringen[7], Kunin[8], Nowacki[9],

* Received date: 2000-05-28; Revised date: 2000-10-20

Foundation item: the National Natural Science Foundation of China (19672023); the International Cooperation project of NSFC (10011130235) and DFG (51520001); the Research Foundation of Liaoning Education Committee (990111001)

Biography: DAI Tian-min (1931 –), Professor, Doctor
Ciarletta and Iesan\cite{10}, and so on.

In literatures on generalized continuum mechanics there exist some principles of work and energy without cross terms, but they all seem to be incomplete. There exists an approach to derive the energy balance law from the principle of work and energy by using equations of motion in micropolar continuum mechanics\cite{7}. There also exist an approach to establish the equations of motion and energy equation for micromorphic continua by subjecting the global energy balance law to invariance under arbitrary constant velocity translation and angular velocity rotation\cite{11}. But these two approaches seem to be unnecessary. There exist some energy balance laws, but they all seem to be incomplete in general. Therefore we should pay great attention to restudying these important basic theoretical problems in generalized continuum field theories.

The purpose of this paper is threefold:

1) To establish new principles of work and energy as well as power and energy rate with cross terms for generalized continuum mechanics;

2) To extend the Piola's theorem of classical continuum mechanics to cases of generalized continuum field theories and to derive all equations of motion, equations of energy and energy rate as well as all boundary conditions for generalized continua in all without additional requirement with the help of the generalized Piola's theorem;

3) To present new balance laws of energy and energy rate for generalized continuum field theories.

For simplicity, we assume that the conservation laws of mass and microinertia hold and neglect the nonlocal residuals of mass and microinertia. For convenience the componental notation is used for derivations and the final results are expressed by symbolic forms, which are suitable for arbitrary coordinate systems. Throughout this paper, all prescribed quantities on boundaries and nonlocal residuals of various fields are denoted by letters carrying a bar "\( \bar{\} \)" and a carat "\(^{\wedge} \)", respectively. Moreover, the Eringen's nomenclatures in \([7,11]\) are used for convenience and comparison.

1 Complete Principles of Power and Energy Rate for Generalized Continua

1.1 Micropolar continua

Let \( \delta \mathbf{v} \) and \( \delta \mathbf{r} \) as well as \( \delta \mathbf{p}^{(n)} \) and \( \delta \mathbf{c}^{(n)} \) be the virtual velocity and virtual angular velocity as well as the virtual stress and virtual couple stress vector. Moreover, let

\[
\begin{align*}
\delta \mathbf{v} = 0 & \quad \text{or} \quad \mathbf{v} = \dot{\mathbf{v}}, \\
\delta \mathbf{r} = 0 & \quad \text{or} \quad \mathbf{r} = \dot{\mathbf{r}};
\end{align*}
\]

on \( \partial R \) (rate boundary), \quad (1a)

as well as

\[
\begin{align*}
\delta \mathbf{p}^{(n)} = 0 & \quad \text{or} \quad \mathbf{n} \cdot \mathbf{t} = \mathbf{p}^{(n)}, \\
\delta \mathbf{c}^{(n)} = 0 & \quad \text{or} \quad \mathbf{n} \cdot \mathbf{m} = \mathbf{c}^{(n)}.
\end{align*}
\]

on \( \partial F \) (force boundary). \quad (1b)

We now postulate the principle of virtual velocity and virtual angular velocity as well as of virtual stress and virtual couple stress with cross terms as follows:

\[
\int_{\Omega} \delta \mathbf{v} \cdot \mathbf{\sigma} \, d\mathbf{v} = \int_{\Omega} \rho (\mathbf{f} - \dot{\mathbf{v}}) \cdot \delta \mathbf{v} \, d\mathbf{v} + \int_{\partial R} \mathbf{p}^{(n)} \cdot \delta \mathbf{v} \, d\mathbf{a} + \int_{\Omega} \rho [ (1 - \mathbf{\sigma}) + \mathbf{x} \times (\mathbf{f} - \dot{\mathbf{v}}) ] \delta \mathbf{r} \, d\mathbf{v} + \int_{\partial R} \left( \mathbf{c}^{(n)} + \mathbf{x} \times \mathbf{p}^{(n)} \right) \cdot \delta \mathbf{r} \, d\mathbf{a},
\]

(2a)