RESEARCH ON THE FLOW STABILITY IN A CYLINDRICAL PARTICLE TWO-PHASE BOUNDARY LAYER*

LIN Jian-zhong, NIE De-ming

(Department of Mechanics, State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, P.R. China)

(Contributed by LIN Jian-zhong)

Abstract: Based on the momentum and constitutive equations, the modified Orr-Sommerfeld equation describing the flow stability in a cylindrical particle two-phase flow was derived. For a cylindrical particle two-phase boundary layer, the neutral stability curves and critical Reynolds number were given with numerical simulation. The results show that the cylindrical particles have a suppression effect on the flow instability, the larger the particle volume fraction and the particle aspect-ratio are, the more obvious the suppression effect is.

Key words: cylindrical particle; two-phase flow; boundary layer; stability

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Introduction

The cylindrical particle two-phase flows are of particular interest in the processing of composite materials, textile industry, paper making, chemical engineering, food processing\[(1)\]. The cylindrical particles in a flow can make the reinforcement of materials, the change of physical property for materials and the reduction of drag. Arranaga\[(2)\] reported that drag reduction effects are up to 60% in pipe flows by adding cylindrical particles to flow. The cylindrical particles have also effects on the mechanisms of flow stability. The effect of cylindrical particle on the flow is related to the viscosity of fluid, the size and the inertia of particle, then cylindrical particle can enhance or suppress the flow stability.

There are much less studies devoted to the effects of cylindrical particles on the mechanisms of flow stability. The flow visualization reported by Pilipenko et al.\[(3)\] represent one of the few available experiments on this subject. In this study, the authors presented results for a cylindrical particle two-phase jet flow at high Reynolds numbers. The addition of cylindrical particles led to an enhancement of the large-scale turbulent structures and a modulation of the turbulence by the

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Biography: LIN Jian-zhong (1958 ~ ), Professor, Doctor (E-mail: mecjzlin@public.zju.edu.cn)
suppression of small-scale structures. However, the effects of cylindrical particles on the mechanisms of stability in a boundary layer are still not reported.

1 The Models and Essential Equations

For a two-dimensional incompressible boundary layer, both fluid and cylindrical particles are treated as continuous medium based on the theory of continuous medium. The basic equations that describe the flow are the continuity and momentum equations:

\[ \nabla \cdot \mathbf{v} = 0, \]

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{\tau}. \]

The stress tensor may be written as the sum of two terms:

\[ \mathbf{\tau} = \mathbf{\tau}^f + \mathbf{\tau}^f = \eta \dot{\gamma} + \eta \phi A (\dot{\gamma} : \mathbf{a}_4). \]

The first term corresponds to the contribution of the fluid, where \( \dot{\gamma} = (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \) is the rate of strain and \( \eta \) the viscosity of the fluid. The second term \( \mathbf{\tau}^f \) represents the contribution of the cylindrical particles, where \( \phi \) is the particle volume fraction. In the semi-dilute regime (\( \phi r \ll 1 \) and \( \phi r^2 \gg 1 \), \( r \) is particle aspect-ratio), we have

\[ A = \frac{r^2}{3 \ln(\sqrt{2 \pi} / \phi)}, \]

where \( \mathbf{a}_4 \) is defined as follows. The constitutive equation is

\[ \frac{\partial \mathbf{a}_2}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{a}_2 - \nabla \mathbf{v} \cdot \mathbf{a}_2 - \mathbf{a}_2 \cdot \nabla \mathbf{v} = \]

\[ \frac{\chi - 1}{2}(\dot{\gamma} : \mathbf{a}_2 + \mathbf{a}_2 : \dot{\gamma}) - \chi (\dot{\gamma} : \mathbf{a}_4) + 2D_r (I - m \mathbf{a}_2), \]

where \( \mathbf{a}_2 \) and \( \mathbf{a}_4 \) are the second-order and the fourth-order orientation tensors of cylindrical particles respectively, \( \chi = (r^2 - 1) / (r^2 + 1) \) is a parameter related to the aspect-ratio of the particles \( r = L/d \), where \( L \) and \( d \) represent the particle characteristic length and thickness, respectively. \( D_r = C_1 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \), \( I \) is the unit tensor and is written as \[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \], here, \( m \) is the dimension of the space and is 2 in the paper.

2 The Equation of Flow Stability

The equation of flow stability, which is expressed with stream function is derived by means of method of the small perturbation. The quantities of flow include base quantity and general quantity, these quantities satisfy the continuity equation and the momentum equation.

Substituting the base quantities in the boundary layer \( U_0(\gamma), V_0(\gamma) = 0 \) and \( \mathbf{v}_0 = FA\phi/Re = HF (F = \dot{\gamma} : \mathbf{a}_4) \) into Eq. (2), we have dimensionless equation:

\[ - \frac{\partial P_0}{\partial x} + H \frac{\partial F_{11}^0}{\partial x} + \frac{1}{Re} U_0 + H \frac{\partial F_{21}^0}{\partial y} = 0, \]

\[ - \frac{\partial P_0}{\partial \gamma} + H \frac{\partial F_{12}^0}{\partial x} + H \frac{\partial F_{22}^0}{\partial y} = 0, \]

where \( F \) is the symmetry second-order tensor.