A NEW METHOD FOR SOLUTION OF 3D ELASTIC-PLASTIC FRICTIONAL CONTACT PROBLEMS *

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Abstract: The solution of 3D elastic-plastic frictional contact problems belongs to the unspecified boundary problems where the interaction between two kinds of nonlinearities should occur. Considering the difficulties for the solution of 3D frictional contact problems, the key part is the determination of the tangential slip states at the contact points, and a great amount of computing work is needed for a high accuracy result. A new method based on a combination of programming and iteration methods, which are respectively known as two main kinds of methods for contact analysis, was put forward to deal with 3D elastic-plastic contact problems. Numerical results demonstrate the efficiency of the algorithm illustrated here.

Key words: 3D frictional contact; elasto-plasticity; programming method; iteration method; the finite element method

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Introduction

Contact problem is of particular interest in numerous engineering applications ranging from structure-structure interaction to machine design and metal forming under various load cases. Contact problems are nonlinear for the moving boundary and the existence of friction along the contact surface. It is a general phenomenon that the nonlinear properties in both material form and contact form will occur at the same time. And the similar properties between the material nonlinearity and contact problems have been demonstrated by laboratory test. Due to the practical importance of the elastic-plastic contact analysis, the problems have been receiving extensive research work over the years. The solution methods in FEA for contact problems can broadly be categorized into two types: incremental iterative method and mathematical programming method.

ZHONG et al. [1] developed a parametric variational principle (PVP) for the analysis of plane contact problems and elastic-plastic structures. In ZHONG’s method [1, 2], both contact and

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elastic-plastic problems can finally be reduced to the same form of parametric programming problem by means of finite element method. The advantages of the method as compared with the conventional ones are that the penalty factors can be canceled and the solutions can be obtained directly without tedious iterative procedures such as general incremental iterative method.

In engineering analysis of a contact problem, a more reasonable way is to take the simulation in 3D modeling form. However, the algorithm used in the 2D analysis is not easy to be extended for the 3D analysis, because for 3D contact problems, generally the algorithm relates to the difficulties of determination of slip direction and too much computing work is needed. The aims of the research work in this paper are to avoid these difficulties.

1 Governing Equations for 3D Contact Analysis

For 3D contact analysis, the governing equations can be concluded as

Equilibrium equations:
\[ d\sigma_{ij,j} + db_i = 0; \]  
(1)

Continuity equations:
\[ d\varepsilon_{ij} = \frac{1}{2}(du_{i,j} + du_{j,i}), \]  
(2)
\[ d\sigma_{ij} = D_{ijkl}(d\varepsilon_{kl} - d\varepsilon_{kl}); \]

Constitutive equations:
\[ f(\sigma_{ij}, \varepsilon_{ij}, \lambda) \leq 0; \]  
(3)
\[ d\varepsilon_{kl} = \frac{\partial g}{\partial \lambda} \lambda, \quad \lambda \geq 0 \quad (f = 0), \]
\[ d\varepsilon_{kl} = 0 \quad (f < 0); \]
\[ n_j d\sigma_{ij} = dp_{ij}, \text{ on } S_{ep}; \]

Boundary conditions:
\[ du_i = d\bar{u}_i, \text{ on } S_u, \]
\[ du_n^{(1)} - du_n^{(2)} + \delta^* \geq 0, \quad p_n \leq 0, \]
\[ p_n(du_n^{(1)} - du_n^{(2)} + \delta^*) = 0, \]
\[ p_r \leq -\mu p_n, \quad |d\bar{u}_r^{(1)} - d\bar{u}_r^{(2)}| = 0, \]
\[ |p_r| = -\mu p_n, \quad |d\bar{u}_r^{(1)} - d\bar{u}_r^{(2)}| \geq 0, \]  
(4)

where the notations of the different variables and parameters are used in the usual way.

The yield condition (3) can be approximately rewritten by the linear term of Taylor’s series:
\[ f^0_a + W_a d\varepsilon - M_a \lambda \leq 0 \quad (a = 1, 2, \ldots, m, \lambda \geq 0), \]  
(5)

where \( f^0_a \) is the initial value of the yield function at instant, \( m \) the number of the yield surfaces, and

\[ W_a = \left( \frac{\partial f_a}{\partial \sigma} \right) D_a, \quad M_a = W_a \frac{\partial g^T}{\partial \varepsilon} \left( \frac{\partial f_a}{\partial \sigma} \right)^T + \frac{\partial f_a}{\partial \lambda} h^T. \]  
(6)

The boundary condition on the elastic-plastic contact surface \( S_c \) (see Fig. 1) can also be written in another form. It will be noticed that for 3D contact problems the expression of Coulomb frictional law is not the same as that in 2D problems, and can be expressed as (see