ON THE RECEPTIVITY OF PIPE POISEUILLE FLOW
WITH A BUMP ON THE WALL UNDER THE PERIODICAL PRESSURE*

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Abstract: Asymptotic method was adopted to obtain a receptivity model for a pipe Poiseuille flow under periodical pressure, the wall of the pipe with a bump. Bi-orthogonal eigen-function systems and Chebyshev collocation method were used to resolve the problem. Various spatial modes and the receptivity coefficients were obtained. The results show that different modes dominate the flow in different stages, which is comparable with the phenomena observed in experiments.

Key words: Poiseuille flow; bi-orthogonal; eigen-function; receptivity

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Introduction

Since the well-known transition experiment of Reynolds (1883)[1], untiring efforts were made to understand its physical mechanism. After more than one hundred years, there is still a lot to learn.

Some process of transition can be taken as five stages: receptivity, linear growth, nonlinear saturation and the break down of the laminar structure, appearance of turbulence. This is an ideal model. Not all the stages will occur in practical situation, and the definition of the stages will not be so clear. But the idea could be helpful for further research. The research of the receptivity is to understand the response of fluid system to external disturbance.

Reynolds' experiment showed that a critical Reynolds number exists. It is a criterion to distinguish the stability or instability of the flow. But the same experimental apparatus will have different orders of magnitude critical Reynolds number under different external condition. For the

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fully-developed parabolic velocity profile, linear theory analysis only has stable results of infinitely small disturbance.

To understand the under-working, attempts were made on such topics. Tatsumi (1952)\(^2\) realized that the onsets of unstable may be related to the inlet boundary conditions, which was confirmed by the succeeding Wygnanski & Champagne (1973)\(^3\) experiment. Many other works indicated that lot of environmental disturbances could cause instabilities. The work here attempted to construct a mathematical and physical model based on such cognition.

Tumin (1996)\(^4\) investigated the receptivity of Poiseuille pipe flow with blow-suction through a narrow gap in the pipe wall. His results showed that the flow near the pipe wall was strongly disturbed. He noted that there were maybe three categories of theoretical models for receptivity problems: 1) The asymptotical analysis of the linearized N-S equations with the Reynolds number tending to infinity; 2) The direct numerical simulation; 3) The numerical methods based on expansion of a solution in spatial eigen-functions of the linear stability problem. In this paper, the last method is adopted.

1 Governing Equations

For axi-symmetrical pipe Poiseuille flow of incompressible viscous fluid, the continuous equation and momentum equations are as follows:

\[
\begin{align*}
\mathbf{V} \cdot \mathbf{V} &= 0, \\
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} &= -\nabla P + \frac{1}{Re} \nabla^2 \mathbf{V},
\end{align*}
\]

where the flow velocity \( \mathbf{V} = (V, U) \), \( P \) is the pressure, \( Re \) is Reynolds number.

Take cylindrical coordinate \( x = (r, z) \), the governing equations become

\[
\begin{align*}
\frac{\partial \tilde{V}}{\partial z} + \frac{1}{r} \frac{\partial (r \tilde{V})}{\partial r} &= 0, \\
\frac{\partial \tilde{U}}{\partial t} + \tilde{U} \frac{\partial \tilde{U}}{\partial z} + \tilde{V} \frac{\partial \tilde{U}}{\partial r} &= -\frac{\partial \tilde{P}}{\partial z} + \frac{1}{Re} \Delta \tilde{U}, \\
\frac{\partial \tilde{V}}{\partial t} + \tilde{U} \frac{\partial \tilde{V}}{\partial z} + \tilde{V} \frac{\partial \tilde{V}}{\partial r} &= -\frac{\partial \tilde{P}}{\partial z} + \frac{1}{Re} \left( \Delta \tilde{V} - \frac{\tilde{V}}{r^2} \right),
\end{align*}
\]

where \( \Delta = \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} \).

For the bump at the wall of the pipe, its shape function can be written as \( K(z) = \epsilon^{1/2} h(z) \) \((\epsilon \ll D, h(z) \sim O(1))\).

The boundary condition on the wall is

\( V = 0, \) on \( r = 1 + K(z), \)

the periodical pressure and the bump are regarded as in the same order of magnitude \((\epsilon^{1/2})\), has the form of \( f(r)e^{-i\omega t} \), where \( \omega \) is the frequency of the pressure, prescribed as a real number. Expand all the quantities in Eqs. (2a) - (2c) in the series as follows:

\[
\begin{align*}
\hat{V} &= \hat{V}(r) + \epsilon^{1/2}(\hat{V}(r,z) + \nu(r;t)) + \\
&\quad \epsilon^{1/2}(\hat{V}(r,z) + \	ilde{\nu}(r,z,t) + \hat{\nu}(r,z;t)) + O(\epsilon^{3/2}) = \\
&\hat{V}(r) + \epsilon^{1/2}(\hat{V}(r,z) + \nu(r)e^{i\omega t}) + \\
&\quad \epsilon(\hat{V}(r,z) + \tilde{\nu}(r,z)e^{i\omega t} + \hat{\nu}(r,z)e^{i(2\omega t)}) + O(\epsilon^{3/2}),
\end{align*}
\]

where \( \hat{V}(r) \) is the basic flow, \( \nu(r;t) \) is the flow corresponding to periodical pressure, \( \hat{V}(r,z) \),