ANALYSIS OF FINANCIAL DERIVATIVES BY MECHANICAL METHOD (Ⅱ)—BASIC EQUATION OF MARKET PRICE OF OPTION *

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Abstract: The basic equation of market price of option is formulated by taking assumptions based on the characteristics of option and similar method for formulating basic equations in solid mechanics: \( h_{0}(t) = m_{1} v_{0}^{-1}(t) - n_{1} v_{0}(t) + F \), where \( h, m_{1}, n_{1}, F \) are constants. The main assumptions are: the ups and downs of market price \( v_{0}(t) \) are determined by supply and demand of the market; the factors, such as the strike price, tenor, volatility, etc., that affect \( v_{0}(t) \) are demonstrated by using proportion or inverse proportion relation; opposite rules are used for purchasing and selling respectively. The solutions of the basic equation under various conditions are found and are compared with the solution \( v_{f}(t) \) of the basic equation of market price of futures. Furthermore the one-one correspondence between \( v_{f} \) and \( v_{0}(t) \) is proved by implicit function theorem, which forms the theoretic base for study of \( v_{f} \) affecting on the market price of option \( v_{0}(t) \).

Key words: option; Black-Scholes formula; differential equation

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1 The Present State, the Significance and the Methods of Option Research

An option is a right of selection. There are two selections for purchasing an option: i.e., to exercise it or not to exercise it. To a buyer, purchasing an option is used for hedging or for making profits from speculation. To a seller, selling an option is used also for hedging or earning premium. Option plays one of the most active roles in modern financial circling\(^{[1]}\). There are many kinds of option and it develops fast. According to the difference of underlying assets (or underlying instrument), option is classified into: currency option; interest rate option; stock/equity option; index option; and commodity option, etc. According to the style of exercise, option is classified into: European style option (option can be exercised only at the maturity date); American style option (option can be exercised at or before the maturity date); exotic option (such as: knock-in option-option is exercised immediately when the price reaches a prescribed level; knock-out option-option is cancelled immediately when the price reaches the

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knock-out level, etc.). According to the evaluation of quotations, option is divided into: call option and put option. According to the place of trade, option is classified into: exchange traded option and OTC (over the counter) option. For an exchange traded option, the strike price, tenor and margin are determined by the exchanges, while for OTC option, these terms are made by the bank and the customer.

Option itself is a zero-sum game. The gain of buyers is equal to the loss of sellers. Option can also do a selling before a purchasing or a purchasing before a selling, it also has leverage.

Option researches include roughly the following contents: the study of option pricing; the analysis of synthetic option; research and developing of new kinds and series of option, etc. As for the study of option pricing, the Black-Scholes option pricing formula is the most famous. The formula makers won Nobel prize for economy in 1997, and thousands of theses on extending B-S model have appeared, which shows that such study has already been the focus of attention.

As for the analysis of synthetic option, it has always been the goal of concern for banking circles, enterprises, financial groups, and foundations. Apart from the traditional methods of tendency forecasting, recently new techniques have been introduced for analysis. For example, C.L. Dunis of the Chemical Bank, London, U.K., used neural network techniques (ANN) for exchange rate forecasting and it seemed a better method than the traditional one.

The research and development of new kinds and series of option, have spread widely thus becoming one of the features of modern finance. For example, in Hong Kong, even medium-size and small bank, such as Zhong Nan Bank, recently offers a series of deposits (premium deposit; option linked deposit) for the choice of investment of the people. From this, we can see that option is not strange to modern people.

This paper studies the changing pattern of market price of European style of option. The price of option obtained by B-S formula is not a real market price of option. A market price of option (premium) is determined by the buying and selling price of public auction (asking and bidding) system carried out by the futures exchanges, i.e., it is determined by supply and demand of the market. Furthermore, the affecting factor volatility in B-S formula is not determined from recent market data but evaluated from historical volatility. Therefore the price of option obtained by B-S formula is not a real market price. In line with the characteristics of option and the factors that affect the price, this paper adopts the method similar to that of formulation of basic equation in solid mechanics to establish the basic equation of market price of option by taking some assumptions. The main assumptions used here are: the ups and downs of the market price of option are determined by supply and demand of the market; factors, such as strike price, tenor, volatility, etc., that affect the price are demonstrated by using proportion or inverse proportion relation; opposite rules are used for purchasing and selling respectively. The established basic equation is a first-order non-linear non-homogeneous differential equation. The solutions of the basic equation under various conditions are found by using the change of variable and separation of variables.

The price of underlying instrument affects on the market price of option. The final part of this paper takes a comparison between the solution of the basic equation of the market price of option and the solution of the basic equation of the market price of futures, and studies the one-one correspondence between the solution and in the general case of solution of market price.