DYNAMIC ANALYSIS OF A SPATIAL COUPLED TIMOSHENKO ROTATING SHAFT WITH LARGE DISPLACEMENTS*

ZHÚ Huái-liáng (朱怀亮)

(Department of Mechanics, Shanghai University, Shanghai 200436, P R China)

Abstract: The dynamic simulation is presented for an axial moving flexible rotating shafts, which have large rigid motions and small elastic deformation. The effects of the axial inertia, shear deformation, rotating inertia, gyroscopic moment, and dynamic unbalance are considered based on the Timoshenko rotating shaft theory. The equations of motion and boundary conditions are derived by Hamilton principle, and the solution is obtained by using the perturbation approach and assuming mode method. This study confirms that the influence of the axial rigid motion, shear deformation, slenderness ratio and rotating speed on the dynamic behavior of Timoshenko rotating shaft is evident, especially to a high-angular velocity rotor.

Key words: Timoshenko rating shaft; dynamic response; nonlinear model; coupled vibration

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Introduction

Rotating shafts are the most vital components of modern industrial and power generation facilities. Due to the importance of these components there were widely studies on the vibration behavior of Euler-Bernoulli rotating shafts using analytical and numerical methods\(^{[1-4]}\). However, together with the high-velocity and flexible shafts are used due to the modern engineering requirements for the higher performance and more complicate circumstances. So more and more studies are to be focused on the dynamic behavior of Timoshenko rating shafts recently. Zu J. W. and Han P. S. investigated the natural frequencies and modes of Timoshenko rating shaft\(^{[5]}\); Choi S.H., Pierre C. and Ulsoy A.G. developed the equations of the flexure vibration of rotating Timoshenko shafts subject to axial loads\(^{[6]}\); Wong E. And Zu J. W. studied the dynamic response of a Timoshenko rotating shaft coupled bending and torsion vibration\(^{[7]}\).

This paper develops the spatial general equations of motion for a Timoshenko rotation shaft with mass unbalance and axially moving. A numerical simulate method is presented for the shaft

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Biography: ZHU Huai-liang (1956 – ), Associate Professor, Doctor (E-mail: zhuhual @163.com)
coupled flexure, torsion and axial vibration, which has a rigid transposition and elastic deformation.

1 Equations of Motion

A typical Timoshenko shaft has the finite length $l$ and is rotating along its longitudinal axis $OX$ in angular velocity $\Omega$. It is subjected to the external torque $M_d$ and axial load $P$ as shown in Fig. 1. Considering the shaft has a uniform circular cross-section and is made of the isotropic material, which can occur large transposition and small elastic deformation.

Let the fixed coordinate system $OXYZ$ be an inertial frame of reference, its unit basis vectors are $a_1$, $a_2$ and $a_3$ respectively. The Cartesian frame $oxyz$ has the unit basis vectors $b_1$, $b_2$ and $b_3$ translating in respect to the axes $OXYZ$. Its axis $ox$ coincides with the undeformed centdc line of the shaft. A disk of infinitesimal thickness of $ds$ is considered, which has a geometric center $c$. The third Cartesian system (moving frame) follows the body of disk to move. The moving frame with unit basis vectors $i$, $j$ and $k$, is defined by the Euler angles $\theta$, $\psi$ and $\varphi$, called roll, yaw, and pitch angles respectively. The matrix transformation is given as

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \quad (1)$$

Assuming that $G$ is the mass center of the disk, $e$ is the unbalance eccentricity, the position and velocity vectors of deformed mass center are given respectively as follows:

$$\begin{aligned} \mathbf{r}_c &= u a_1 + v a_2 + w a_3 + e \cos\theta j + e \sin\theta k, \\
\dot{\mathbf{r}}_c &= \dot{u} a_1 + \dot{v} a_2 + \dot{w} a_3 + e \cos\theta \mathbf{\omega} \times j + e \sin\theta \mathbf{\omega} \times k, \quad (2) \end{aligned}$$

where ($'$) is the time derivative with respect to time, $\mathbf{\omega}$ is the angular velocity of moving frame, it may be denoted by

$$\mathbf{\omega} = \omega_1 i + \omega_2 j + \omega_3 k, \quad (4)$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\varphi \\ 0 & \cos\theta & \sin\theta \cos\varphi \\ 0 & -\sin\theta & \cos\theta \cos\varphi \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\varphi} \end{bmatrix}. \quad (5)$$

Hence, the velocity of the mass center is given as