AN ALGEBRAIC MULTIGRID METHOD FOR COUPLED THERMO-HYDRO-MECHANICAL PROBLEMS *

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Abstract: An algebraic multigrid method is developed to solve fully coupled multiphase problem involving heat and mass transfer in deforming porous media. The mathematical model consists of balance equations of mass, linear momentum and energy and of the appropriate constitutive equations. The chosen macroscopic field variables are temperature, capillary pressure, gas pressure and displacement. The gas phase is considered to be an ideal gas composed of dry air and vapour, which are regarded as two miscible species. The model makes further use of a modified effective stress concept together with the capillary pressure relationship. Phase change is taken into account as well as heat transfer though conduction and convection and latent heat transfer (evaporation-condensation). Numerical examples are given to demonstrate the computing efficiency of this method.

Key words: multiphase flow; deforming porous media; phase change; grid; iteration

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Introduction

In recent years, a great deal of attention has been directed towards full coupled thermo-hydro-mechanical analysis in deforming porous media due to extraction of underground fluids (water, petroleum, natural gas) in reservoir, environment and construction engineering (Lewis and Schrefler 1998; Gawin, 1995)[1,2]. Multiphase flow analyses are computationally intensive because they involve problems of nonlinear, regional size and very long time spans. Geometric multigrid approach has been developed to solve partial differential equations in the 80s of the 20th century, and used well into fluid mechanics. Multigrid iteration schemes rely upon using a hierarchy of grids (from fine to coarse) to solve a set of discrete equations. The short wavelength errors can be very effectively reduced by solving the equations of the original problem on the finest grid, then go onto coarser grids to remove the long wavelength errors by solving the...
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residual equations. With each grid proving particularly effective for removing errors of wavelength characteristic of the mesh spacing on that grid, very high computing efficiency can be obtained. However, discretization on an unstructured or finite element mesh demands a multigrid solver capable of automatically generating its own hierarchy of coarser grids. In recent years, algebraic multigrid method (AMG) has been developed to do just this, making use only of the equation matrix itself to create the coarse grids (Webster, 1994)[3]. AMG is relatively economical on computer storage and CPU time, both scaling linearly with problem size. In this paper, this algorithm is extended to solve the fully coupled multiphase problem involving heat and mass transfer in deforming porous media.

The basic formulation of the physical problem solved is first briefly described, then the details of the computing strategy. Numerical examples are presented to demonstrate the effectiveness of the method.

1 Numerical Model for Thermo-Hydro-Mechanical Problem

The fully coupled numerical model to simulate slow transient phenomena involving flow of heat, water and gas in deforming porous media can be described as follows. The interconnected pores are partly filled with water and partly with gas, which is a mixture of dry air and water vapour. Dry air, vapour and their mixture are assumed to behave as ideal gases. Phase change is considered only for the fluid phases. The chosen macroscopic field variables are displacement $u$, capillary pressure $p_c$, gas pressure $p_g$ and temperature $T$.

1.1 Constitutive relations

The degree of saturation $S_w$ is a function of capillary pressure $p_c$ and $T$, which is determined by experiment

$$S_w = S_w(p_c, T).$$

The capillary pressure $p_c$ is defined as

$$p_c = p_g - p_w,$$

where $p_w$ is pressure of liquid water. The moist gas ($g$) in the pore system is assumed to be a perfect mixture of two ideal gases, dry air ($g_a$) and water vapour ($g_w$). The equation of state of perfect gas is valid

$$p_{g_a} = \rho_{g_a} TR / M_{g_a}, \quad p_{g_w} = \rho_{g_w} TR / M_{g_w}, \quad p_g = p_{g_a} + p_{g_w},$$

where $M_{g_a}$, $\rho_{g_a}$ are the molar mass and density of constituent $g_a$ (dry air and water vapour) and $R$ the universal gas constant, $T$ absolute temperature. Further Kelvin-Laplace law applies, giving the relative humidity $H$ of the moist air inside the pores as

$$H = \frac{p_{g_w}}{p_{g_{sw}}} = \exp \left( - \frac{p_c M_w}{\rho_w RT} \right).$$

The water vapour saturation pressure $p_{g_{sw}}$ is obtained from the Clusius-Clapeyron equation

$$p_{g_{sw}}(T) = p_{g_{sw0}} \exp \left[ - \frac{M_w \Delta h_{vap}}{R} \left( \frac{1}{T} - \frac{1}{T_0} \right) \right],$$

where $T_0$ is the reference temperature, $p_{g_{sw0}}$ is water vapour saturation pressure at $T_0$ and $\Delta h_{vap}$ the specific enthalpy of evaporation.

The constitutive laws of the solid phase are introduced through the concept of modified