PERIODICAL INTERFACIAL CRACKS IN ANISOTROPIC ELASTOPLASTIC MEDIA *

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Abstract: By using Fourier transformation the boundary problem of periodical interfacial cracks in anisotropic elastoplastic bimaterial was transformed into a set of dual integral equations and then it was further reduced by means of definite integral transformation into a group of singular equations. Closed form of its solution was obtained and three corresponding problems of isotropic bimaterial, of a single anisotropic material and of a bimaterial of isotropy-anisotropy were treated as the specific cases. The plastic zone length of the crack tip and crack opening displacement (COD) decline as the smaller yield limit of the two bonded materials rises, and they were also determined by crack length and the space between two neighboring cracks. In addition, COD also relates it with moduli of the materials.

Key words: periodical crack; interfacial crack; anisotropic elastoplastic fracture of bimaterial; antiplane problem; Dugdale-Barenblatt (D-B) model; crack opening displacement (COD)

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Introduction

Quite a lot of works have discussed the problems of collinear crack in either isotropic material\(^1\) or anisotropic material\(^2\), however they are usually restricted to linear elasticity. Analysis for elastoplastic problem of collinear interfacial cracks of bimaterial has not been reported yet. It seems that the methods for solving fracture problems of single material are rarely capable of untangling fracture problems of bimaterial. Recently Ref.\(^3\) succeeded in solving an elastoplastic problem of a single crack by virtue of cosine-transformation.

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This paper presents an investigation on anisotropic elastoplastic antiplane problem of collinear periodical interfacial fracture. The periodical boundary problem is interpreted into a set of integral equations by employing Fourier Transformation. Later close form of the solution is derived from the integral equations based on Definite Integral Transformation and the theory of integral equation. Finally, from general to specific, the results for the problems of isotropic-anisotropic bimaterial, of isotropic-isotropic bimaterial and of a single anisotropic material are obtained. The method developed in the present paper may also fit to in-plane problem.

1 Description of Problem

As shown in Fig. 1, the upper and the lower plane are occupied by No. 1 and No. 2 medium, respectively. The x-axis of Cartesian coordinate system $Ox_jz_j$ is attached on the interface of the two media. The origin $O$ and x-axis are adopted by both media, while $y_j$-axis points into No. $j$ medium. Lying at the interface are the periodical cracks with length of $2a$ and space of $2h$. At the places of $y_j \rightarrow \infty$, shear loads $\tau_0$ are applied.

![Fig. 1 Geometry and coordinate system of an infinite bimaterial embedded with periodical interfacial cracks spacing $2h$](image)

Applying Dugdale-Barenblat (D-B) model and noting the plastic zone length of the crack tip to be $b-a$, we simplify the original problem into a boundary problem of which on the extended cracks long $2b$ are loaded with antiplane shear stresses and at infinites ($y_j \rightarrow \infty$) is loaded with no force.

The equilibrium differential equations in terms of $z_j$-displacement $w_j$ are

$$C_{55j} \frac{\partial^2 w_j}{\partial x^2} + 2C_{45j} \frac{\partial^2 w_j}{\partial x \partial y_j} + C_{44j} \frac{\partial^2 w_j}{\partial y_j^2} = 0,$$

where $C_{44j}, C_{45j}, C_{55j}$ are material constants, which are given. The subscript $j$ represents material types.

Boundary conditions are

$$C_{44j} \frac{\partial w_j(x,0)}{\partial y_j} + C_{45j} \frac{\partial w_j(x,0)}{\partial x} = -\tau_0 \quad (x \in (2mh - a, 2mh + a)), \quad (2)$$