EQUILIBRIUM WITH ENDOGENOUS TECHNOLOGICAL CHANGES: THEORY AND APPLICATIONS

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In this paper we use a continuous-time general equilibrium model to analyze the problem of evaluating new irreversible investment opportunities that take the form of technological changes. Being a new technology, by definition, not perfectly correlated with the existing ones, the traditional spanning assumption invoked by the Real Option literature is not applicable in this context and a general equilibrium approach provides a more suitable framework. We analyze the problem of optimal consumption and investment of the representative individual in the context of the Cox Ingersoll and Ross (1895) model. We characterize the solution of the model under different assumptions about the utility function of the representative individual and about the parameters describing the technologies in the economy. A major result obtained from the model is the possibility of jumps in the equilibrium rate of return at the time in which technological changes are implemented. This is particularly interesting considering the fact that such jumps are obtained endogenously from the solution of the optimization problem and not imposed by "ad hoc" assumptions on the evolution of some variables.

1. Introduction

As it is broadly stressed in Dixit and Pindyck (1994), investment decisions depend on three important characteristics in varying degrees: irreversibility, uncertainty and timing. The interaction of these three variables in the determination of optimal investment decisions is the focus of the growing literature on "real options". According to this view, whose origins can be dated back to Myers' (1977) pathbreaking paper, the new investment opportunity is seen as an American option on the investment. Maintaining the option alive gives the holder the opportunity to acquire better knowledge on the resolution of uncertainty; exercising it implies paying a (partial) sunk cost to undertake the project. Accordingly, uncertainty, timing and irreversibility play major roles in the decision to invest, which can be captured in an "option-like" approach to capital budgeting decisions.

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As an alternative to the traditional NPV rule, the Real Option Approach to investment provides a better description of the sensitivity of investment opportunities to the cost of capital. Empirical analysis (Summers (1987), and Dertouzas et al. (1990)) suggests that investment opportunities are highly sensitive to uncertainty over the future value of the project. Changes in economic conditions that affect the perceived riskiness of future cash flows can have a larger impact on investment spending than a change in the interest rate.

A key feature of the real option approach to investment, as it is made explicit in Brennan and Schwartz (1985), McDonald and Siegel (1986) and Dixit and Pindyck (1994) is an assumption about completeness of financial markets. These models, in fact, assume that the uncertain output deriving from an investment opportunity can be “spanned” by the existing securities in the market and therefore the new investment can be valued by using standard no-arbitrage techniques. In the case of a new production technology this assumption appears too strong. A new production technology is, by definition, not perfectly correlated with any existing one and therefore the principle of no-arbitrage evaluation is inappropriate in this context. As it is explicitly pointed out by Schwartz (1994):

"[...] if the underlying uncertainty comes from traded assets or can be spanned by traded assets, the arbitrage methods [...] should be used. Otherwise, the drift of the stochastic process for the state variables should be risk adjusted using an equilibrium model."

This is the inspiring idea for this paper. Following previous work by Wang (1995) we will develop a general equilibrium model in continuous-time in which we introduce irreversible investment opportunities in the form of technological changes. The framework for our analysis is provided by the general equilibrium model of Cox, Ingersoll and Ross (1985a) (CIR henceforth). It is important to recognize that, without the completeness assumption, the introduction of the new products of the investments affects, on one side, the aggregate supply in the economy and, on the other side, changes the choice sets of individuals. A major consequence is therefore that the consumption-portfolio decision of the representative agent in a general equilibrium model cannot be kept separate from the investment decision in the new technology. Recognizing this interaction allows us to correctly formulate the problem that the representative individual faces and to provide equilibrium characterizations of return process and price process of financial claims. The traditional consumption-investment problem in this context acquires a new decision variable, namely the time to switch to the new technology. As a result, the description of the economy is characterized by two separate economies pasted together at the time of technological change. Wang proved that, although the equilibrium consumption and price processes are smoothly pasted together, the possibility of technological switches may cause the equilibrium return process to be discontinuous at the time of technological changes.

In this paper we focus our attention on the characterization of the switching time when the preferences of individuals are described by isoelastic (Constant Relative Risk Aversion) utility functions. We find that, with non-stochastic technology parameters, the switching time is a deterministic variable determined as a solution of an optimal stopping problem. The equilibrium rate of return, in the case of CRRA utility functions is in general discontinuous at the time