MATHEMATICAL MODELING OF PRIMARY PRODUCTION AND DESTRUCTION OF PHYTOPLANKTON IN OPERATING AND PLANNED RESERVOIRS

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Along with the entry of allochthonous organic matter with river and waste waters into a reservoir, the new formation of autochthonous organic matter occurs during photosynthesis of algae, in particular, phytoplankton. Depending on the size, level of pollution, and trophic state of the reservoir, primary production of phytoplankton during the growing season can be both severalfold less and greater than the corresponding inflow of allochthonous organic matter. Accordingly, its effect on water quality will be less or more considerable.

Phytoplankton consists of algae having many tens of species and represented by small microscopic forms whose lifespan in the overwhelming majority of cases is measured in days or even hours. However, these algae are capable of rapid multiplication, as a result the annual phytoplankton production exceeds by tens of times its average and even maximum biomass, and the elements composing the phytoplankton in accordance with this turn over tens of times a year. Molecular oxygen evolved during photosynthesis of phytoplankton serves as a source of replenishing its store in water, comparable in value to atmospheric aeration. On the other hand, destruction (mineralization) of phytoplankton is accompanied by the uptake of oxygen dissolved in water and the liberation of breakdown products, including toxic substances (hydrogen sulfide, phenols, cyanides), which in the case of mass death of phytoplankton can lead to pronounced depreciation of water quality and to fish-kill phenomena due to an oxygen deficit and intoxication of fish and other representatives of useful fauna. Such conditions can occur in a reservoir at places of the accumulation of large masses of phytoplankton as a result of their being driven together by wind-driven currents.

Consequently, under certain conditions phytoplankton can considerably affect water quality and the oxygen regime of a reservoir. Therefore calculation of primary production and destruction of phytoplankton is one of the most important elements of calculating the water quality and oxygen regime of a reservoir.

The model of primary production and destruction of phytoplankton of a reservoir, which is based on the simulation mathematical model of the phosphorus system presented in [1], can be written in the following form:

\[
\Phi_s = K_F F \Delta t \Delta W \frac{n}{1 + F/\beta \cdot DIP} ;
\]

(1)

\[
\Delta \Phi = K_B B \Delta t W \frac{n}{1 + F/\beta \cdot DIP} ;
\]

(2)

\[
K_F = K_{TF} R_{TF} R_{IF} \frac{1}{1 + F/\beta \cdot DIP} ;
\]

(3)

\[
R_{TF} = 0.2 + \frac{0.022 \left( \frac{0.217 \Delta t}{t} - 1 \right)}{1 + 0.028 e^{0.217 \Delta t}} ;
\]

(4)

\[
R_{IF} = (e/\kappa \cdot h)[\exp(-r_s) - \exp(-r_t)] ;
\]

(5)

\[
r_t = l_{av}/l_{opt} ;
\]

(6)

\[
r_s = r_t \cdot \exp(-k h) ;
\]

(7)

\[
k_t = k_a + k_b \cdot \text{Chl} a ;
\]

(8)
where \( \Phi_i \) and \( \Phi_i \) are respectively gross primary production and destruction of phytoplankton during the \( i \)-th month of the growing season, expressed in \( \text{C}_\text{org} \) units; \( F \) and \( B \) are the average concentration of respectively phytoplankton and bacterioplankton during the growing season, expressed in phosphorus units; \( \Delta t \) is the duration of the \( i \)-th month of the growing season (days); \( n \) is the conversion factor from phosphorus to \( \text{C}_\text{org} \); \( K_F \) and \( K_B \) are respectively the average monthly rate of consumption of dissolved inorganic phosphorus (DIP) by phytoplankton and dissolved organic phosphorus (DOP) by bacterioplankton; \( K_1 \) is the maximum rate of consumption of DIP by phytoplankton; \( R_{TF} \) and \( R_{TB} \) are the average monthly coefficients for correcting \( K_1 \) for water temperature and illumination (dimensionless characteristics); \( \beta \) is the coefficient of conversion of the substrate (DIP) by phytoplankton (dimensionless quantity); \( T_1^\circ \) is the average monthly water temperature in the photosynthetic zone of the reservoir; \( \theta_1^\circ \) is the average monthly water temperature in the reservoir (average over the depth); DIP and DOP are respectively the average concentration of dissolved inorganic and organic phosphorus during the growing season; \( e \) is the base of natural logarithms; \( h \) and \( \Delta W \) are the average depth and volume of the photosynthetic zone of the reservoir during the growing season; \( I_{av} \) is the average monthly illumination; \( I_{opt} \) is the optimal value of illumination; \( k_1 \) is the coefficient of extinction of illumination with depth; \( k_a \) and \( k_b \) are constants; Chl \( a \) is the average concentration of chlorophyll \( a \) during the growing season; \( \alpha \) is the stoichiometric coefficient; \( K_2 \) is the maximum rate of consumption of DOP by bacteria; \( R_{TB} \) is the average coefficient for correcting \( K_2 \) for water temperature; \( W_i \) is the average volume of the reservoir during the \( i \)-th month of the growing season.

For an operating reservoir the quantities \( F \), \( B \), \( \beta \cdot \text{DIP} \), and DOP for known values of gross primary production \( \Phi_g \) and destruction \( \Phi_i \) of phytoplankton during the growing season are found by solving the inverse problem — by numerical solution of the system of equations:

\[
1 + \frac{F}{\beta \cdot \text{DIP}} \approx \frac{\gamma F}{1.7 + 18.723 F},
\]

\[
1 + \frac{B}{\text{DOP}} = pB,
\]

\[
y = \frac{\phi B}{R_{TB} \Delta W e h \cdot n},
\]

\[
\rho = \frac{\phi B}{R_{TB} \Delta W n},
\]

\[
z = \exp \left( -\frac{I_{opt}}{I_{opt}} \right) - \exp \left( -\frac{I_{av}}{I_{opt}} \right);
\]

\[
x = \frac{\phi \cdot R_{TF} \Delta W e}{13.33 + \phi F},
\]

\[
y = \frac{\phi \cdot R_{TB} \Delta W e}{13.33 + \phi F};
\]

where \( W \) is the average reservoir volume during the growing season; \( R_{TF} \) and \( R_{TB} \) are the average coefficients for correcting \( K_1 \) and \( K_2 \) for water temperature during the growing season; \( \Phi_g = \phi \); \( D \) is the duration of the growing season (days); \( I_{av} \) is the average illumination during the growing season.

For a planned reservoir the quantities \( F \) and \( \beta \cdot \text{DIP} \) can be found by numerical solution of the system of equations:

\[
\frac{F}{\beta \cdot \text{DIP}} = \frac{F}{\beta \cdot \text{DIP}} = \frac{h_o}{\phi},
\]

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\[
\frac{h_o}{\phi}.
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