Radial decline of the extracellular action potential

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Abstract—A modified line source model presented earlier has been used to study the decline of the extracellular single muscle fibre action potential. The muscle tissue is modelled as a low-pass filter. The transfer function of the filter declines more slowly than a first-order low-pass filter at low frequencies, but much faster at high frequencies. The cutoff frequency of the filter increases when the anisotropy of the muscle decreases. It also increases proportionally with the propagation velocity of the action potential. The decline of different frequency components obtained from the modified line source volume conductor and a filter model derived from experimental measurements are compared and their differences explained. The modified line source model was found to be identical to the volume conductor model in terms of results and at the same time conceptually simple for applications.

Keywords—Action potential, Computer simulations, Electrical characteristics, Low-pass filters, Radial decline


1 Introduction

NANDEKAR and STALBERG (1983a) have presented a modified line source model to compute the extracellular single muscle fibre action potential. The potential can be expressed as a convolution of the transmembrane current $i(t)$ and a weight function $h(t)$

$$p(t) = i(t) * h(t)$$

where $*$ denotes the convolution. Using the Fourier transform on eqn. 1 we obtain

$$P(j\omega) = I(j\omega)H(j\omega)$$

(2)

where the upper case symbols represent the Fourier transform and $\omega$ is the frequency variable. The weight function is computed as the potential recorded by the electrode when a unit current source travels from the end-plate to the tendons at the speed of the propagation velocity of the action potential in the muscle fibre. The computation of this function for different recording surfaces has been described by NANDEKAR and STALBERG (1983a, b).

Using simulation techniques, we have modelled the muscle tissue as a first-order low-pass filter as described by GATH and STALBERG (1977). The decline of different frequency components of the action potential is investigated. The results are compared with previously reported studies using a volume conductor model (ROSENFALCK, 1969) and multielectrode recordings of single muscle fibre action potential (GATH and STALBERG, 1977). Their similarities and differences are discussed in order to determine the usefulness of the modified line source model.

2 Methods

GATH and STALBERG (1977) used a 14-lead multielectrode to measure the action potential from the muscle fibre. By measuring the action potential from two leading-off surfaces, 25 $\mu$m in diameter, they computed the transfer function of the tissue, 76 $\mu$m thick, separating them. The method is described schematically in Fig. 1. The tissue was modelled as a first-order low-pass filter with a transfer function of the form

$$T(j\omega) = \frac{P_2(j\omega)}{P_1(j\omega)} = \frac{A}{1+j\omega\tau} \quad . \quad . \quad . \quad (3)$$

where $A$ is the gain factor and $\tau$ is the time constant. The average values of these parameters were found to be 0.55 and 69 $\mu$s, respectively.

From eqn. 2, the two potentials can be expressed as

$$P_1(j\omega) = I(j\omega)H_1(j\omega) \quad . \quad . \quad . \quad (4a)$$

$$P_2(j\omega) = I(j\omega)H_2(j\omega) \quad . \quad . \quad . \quad (4b)$$
Hence the transfer function of the tissue can also be computed as

\[ T(j\omega) = \frac{H_2(j\omega)}{H_1(j\omega)} \quad \ldots \ldots \ldots \quad (5) \]

We have duplicated the procedure summarised in Fig. 1 by simulation and used eqn. 5 to compute the transfer function.

\[ \begin{align*}
\text{Muscle Fibre} & \quad \text{Multi-electrode} \\
\text{Schematic of the technique used by GATH and STALBERG (1977) to compute the transfer function of the muscle tissue. The tissue is modelled as a filter. The potential measured by the closer electrode is assumed to be the input and the potential measured by the further electrode to be the output of the filter.}
\end{align*} \]

The weight function was computed as described by NANDEDKAR and STALBERG (1983a). It depends on the following factors.

(a) **Propagation velocity**
A linear relationship between fibre diameter (\(\mu m\)) and propagation velocity (\(m s^{-1}\)) was used (NANDEDKAR and STALBERG, 1983a)

\[ \text{velocity} = 3.7 + 0.05 (\text{diameter} - 55) \quad \ldots \ldots \ldots \quad (6) \]

(b) **Position of the electrode**
GATH and STALBERG (1977) positioned one recording surface on the multielectrode close to the fibre and recorded potentials from two surfaces that were 76 and 152 \(\mu m\) away. The distance between the fibre and the closest recording surface was not known. We arbitrarily assumed this distance to be 10 \(\mu m\). Hence the weight functions were computed for recording positions 10, 86 and 162 \(\mu m\) away from the fibre surface.

(c) **Anisotropy**
The potential of a unit current source in a medium with cylindrical anisotropy is

\[ \Phi(r, z) = \frac{1}{4\pi\sigma_r\sqrt{Kr^2 + z^2}} \quad \ldots \ldots \ldots \quad (7) \]

where \((r, z)\) are the co-ordinates of the electrode with respect to the current source; \(K\) is the anisotropy (ratio of the axial to radial conductivity) and \(\sigma_r\) is the radial conductivity of the muscle tissue. We assume that the anisotropy is constant at all frequencies. There is great variability in the electrical conductances reported in the literature (GEDDES and BAKER, 1967). In the present computations, \(\sigma_r\) acts only as a scale factor and hence may be neglected. Anisotropy, on the other hand, will affect the filter characteristics.

A sampling interval of 25 \(\mu s\) was used to calculate records of the weight function containing 1024 samples. The Fourier spectra of these records have a resolution of 40 Hz and cover frequencies up to 20 kHz.

The weight function is the potential measured by the electrode when a unit current source travels from the endplate to the tendons. It increases as the source approaches the electrode, reaching a maximum when they have no axial separation and then decreases as the source moves away (NANDEDKAR and STALBERG, 1983a). The weight function computed for two different electrode positions is shown in Fig. 2a. For clarity, an interval of 1 ms where the functions differ significantly is shown.

\[ \begin{align*}
\text{Fig. 2} & \quad (a) \text{Weight function computed for two recording positions of the electrode. Only a short interval where they differ significantly is shown. The propagation velocity is 4 m s}^{-1}. \\
& \quad (b) \text{Transfer function of the tissue computed from the weight functions in (a) compared with a low-pass filter (C) with unit gain and the same -3 dB frequency. The transfer function is normalised so that the gain at the fundamental frequency is unity.}
\end{align*} \]

Note that the potential recorded by the electrode at both positions is almost identical when the source is away from the electrode. This portion of the record contains the low-frequency components. However, when the source is close to the electrode, the two functions differ significantly. This portion of the signal contains the high-frequency components. It can easily be seen that when moving the electrode away from the fibre the low frequencies are relatively less affected compared with the high frequencies. The muscle tissue can therefore be modelled as a low-pass filter.

The transfer function of the tissue obtained from eqn. 5 is shown in Fig. 2b. The transmembrane current used in simulations has no DC component, and so the simulated action potentials also do not have a DC component. Therefore the gain of the filter was defined as the gain measured at the fundamental frequency component of the spectrum. The frequencies where the gain declined by 3 dB and 20 dB were measured. A Bode plot of a low-pass filter with unit gain and the same -3 dB frequency is compared with the computed muscle filter characteristic. The transfer function declines more slowly than a low-pass filter at low frequencies, while the decline is much faster at high frequencies.

The filter characteristics can also be studied by measuring the decline of individual frequency components with the radial distance of the electrode from the fibre. The amplitude of the frequency components in the weight function was computed when the electrode was moved away from the fibre until it was 10 per cent of its value when measured 5 \(\mu m\) from the surface. A similar approach was used by ROSENFALCK (1969) to study the decline of different frequency components in the muscle. GATH and STALBERG (1977) used the transfer function in eqn. 3 to estimate this distance. The transfer function of a muscle tissue with thickness \(r\) was computed as

\[ T(j\omega) = (T_{0}(j\omega))^r_{76}. \quad \ldots \ldots \ldots \quad (8) \]