DYNAMIC MODEL OF THE OXYGEN REGIME OF A RESERVOIR

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At present there is no generally accepted model of the oxygen regime of reservoirs similar to that developed for streams by Streeter and Phelps and which served as the basis for its numerous modifications [1, 2]. In the domestic literature investigations of these problems are limited, as a rule, just to a mathematical formulation of a model of the oxygen regime of water bodies (lakes and reservoirs) without their application to a particular object. However, in the foreign literature a comparatively small number of models of the oxygen regime have been brought to the stage of their practical application to particular water bodies, to reservoirs in particular [1].

The model of the oxygen regime of a reservoir proposed in the present work in a theoretical respect represents an attempt to generalize, systematize, and modify a number of existing models in various areas of science: hydrology, hydrochemistry, and hydrobiology as applied to the problem of calculating and predicting the oxygen regime of operating and planned reservoirs. It is based on the aforementioned Streeter—Phelps model and includes in essence two independent models: a model of the reservoir water quality (so-called ideal mixer model) [3] and model of primary production and destruction of reservoir phytoplankton [4]. The practical application of this model is illustrated by examples of calculating and predicting the oxygen regime of an operating and planned reservoir.

As is known, the content of oxygen dissolved in water depends on the intensity of processes providing entry of oxygen in water and causing its consumption. The income part of the oxygen balance is made up of the entry of oxygen from the atmosphere and higher aquatic vegetation during photosynthesis. Both processes, leading to enrichment of the water in oxygen, occur mainly in the upper water layers.

The outgo of dissolved oxygen occurs due to processes of oxidation of organic matter during respiration and decomposition of plants and animals as well as by its release into the atmosphere. Biochemical processes of destruction of organic matter occur in the entire water column and in bottom sediments. The temperature, regulating the solubility of oxygen in water, determines its absolute content.

The proposed model permits calculating the income and outgo components of the oxygen balance of reservoirs under conditions of their water, ice, thermal, hydrochemical, hydrobiological, and meteorological regimes and anthropogenic load varying during the year. It differs, in particular, in this from the steady-state Streeter—Phelps model, which can be used for calculating the oxygen regime of streams only under conditions of a steady regime. The difference of these models is determined mainly by the fact that they are based on different hypotheses of the movement of water in a river and reservoir.

The Streeter—Phelps model can be written in the following form:

\[ D_e = \frac{K \cdot \text{BOD}_s \Delta t}{k - K} \left( e^{-k \tau} - e^{-k \tau} \right) + D_s e^{-k \tau} \]  \hspace{1cm} (1)

where \( D_e \) is the oxygen deficit at the end of the calculated stretch of the river and end of the calculated time interval \( \Delta t \); \( D_s \) is the oxygen deficit at the start of the calculated stretch of the river and start of the calculated time interval \( \Delta t \); \( \text{BOD}_s \) is the content of readily oxidized organic matter in river water at the start of the calculated stretch of the river and start of the calculated time interval \( \Delta t \) expressed in units of total BOD; \( \tau \) is the travel time of water in the calculated stretch of the river; \( e \) is the base of natural logarithms; \( k \) is a constant characterizing the rate of oxidation of organic matter (water self-purification coefficient); \( K \) is a constant characterizing the rate of saturation of water by atmospheric oxygen (reaeration coefficient).

Obviously, underlying the Streeter—Phelps model is the model of a so-called ideal displacer, which can be written in the following form:

\[ \text{BOD}_t(D_e) = \text{BOD}_t(D_s) e^{-k \tau} \]  \hspace{1cm} (2)

Formula (2) characterizes the process of oxidation of organic matter in river water and process of saturation of water by atmospheric oxygen, which occur independently of each other. Actually, both these processes are interdependent in nature and occur simultaneously, which is taken into account in the Streeter–Phelps model.

In connection with this the Streeter–Phelps model can be applied only to rivers where the movement of water, and pollutants (including dissolved oxygen) along with it, can tentatively be regarded as a process of displacement of "old" water and its replacement by "new" in the manner of piston displacement. Various modifications of the Streeter–Phelps model, a review of which is given, in particular, in [1, 2], are in principle also based on an ideal displacer model and therefore are applicable only to rivers.

Meanwhile, the process of movement of water in a reservoir is characterized not only by the displacement of "old" water by "new" but simultaneously also by their constant mixing, which is reflected in the ideal mixer model [3]. Therefore, the replacement of the ideal displacer model by this model in formula (1), obviously, permits applying the Streeter–Phelps model also to reservoirs. In this case the modified Streeter–Phelps model takes on the following form:

\[ D_e = \frac{x}{k - x} \left[ \text{BOD}_0 \left(1 - B\right) + \frac{P_{\text{BOD}}}{V_{\text{in}} + V_{\text{pr}} + V_{\text{ev}}} - \left(1 -\alpha\right)V_{\text{ev}} + kW\Delta t \right] - \left[ \text{BOD}_0 \left(1 - B'\right) + \frac{P_{\text{BOD}}}{V_{\text{in}} + V_{\text{pr}} + V_{\text{ev}}} - \left(1 -\alpha\right)V_{\text{ev}} + kW\Delta t \right] \]

\[ D_s = \left(\alpha_2 - \frac{C_{\text{W}} + P_{\text{pr}}O_2 + P_{\text{ww}}O_2 + P_{\text{pr}}O_2}{W_{\text{in}} + V_{\text{in}}} \right) \left( B'V_{\text{in}} + V_{\text{pr}} + V_{\text{ev}} - \left(1 -\alpha\right)V_{\text{ev}} + kW\Delta t \right) \]

where \( D_e \) and \( D_s \) is the average oxygen deficit of the water body at the end and start of the calculated time interval \( \Delta t \) (month); \( \text{BOD}_0 \) is the total BOD at the start of the interval \( \Delta t \) determined by the ideal mixer model given in [3]; \( B \) and \( B' \) are the average water renewal coefficient during time \( \Delta t \), determined by the formulas given in [3], respectively, with consideration of coefficient \( K \) and \( k \); \( K \) is the average integral water self-purification coefficient during time \( \Delta t \), determined for total BOD by the method given in [3, 5]; \( k \) is the average reaeration coefficient during time \( \Delta t \); \( P_{\text{BOD}} \) is the total inflow of dissolved allochthonous and autochthonous organic matter in the reservoir during time \( \Delta t \), expressed in units of total BOD and determined by the method given in [3, 5]; \( P_{\text{pr}}O_2 \) and \( P_{\text{ww}}O_2 \) are the inflow of oxygen into the reservoir during time \( \Delta t \) with river waters and wastewaters; \( P_{\text{pr}}O_2 \) is the difference between the inflow of oxygen into the water body due to photosynthesis of algae and consumption of oxygen on their destruction (mineralization) during time \( \Delta t \), in other words, net photosynthesis, expressed in oxygen units and determined by the model given in [4]; \( k_d \) is the average rate of oxygen consumption by bottom deposits during time \( \Delta t \); \( F \) is the area of the bottom deposits; \( O_2 \) is the solubility of oxygen at the average surface water temperature of the water body during time \( \Delta t \), in other words, the saturation concentration, determined by formula (4):

\[ O_2^S = 14.61996 - 0.4042T + 0.00842T^2 - 0.00009T^3 \]

where \( T \) is the water temperature; \( C_s \) is the concentration of oxygen dissolved in water at the start of the calculated time interval, determined as the difference between the corresponding saturation concentration and oxygen deficit by the formula

\[ C = O_2^S - O_2 \]

where \( W_{\text{in}} \) is the volume of liquid water in the reservoir at the start of the time interval \( \Delta t \); \( V_{\text{in}} \) is the volume of inflow of water into the reservoir with river and wastewaters during time \( \Delta t \); \( V_{\text{pr}} \) is the volume of solid or liquid precipitation that fell on the reservoir surface during time \( \Delta t \); \( V_{\text{ev}} \) is the volume of water that evaporated from the reservoir surface during time \( \Delta t \); \( V_{\text{in}} \) is the volume of inflow (outflow) of liquid water of the reservoir during time \( \Delta t \) related to melting (formation) of ice; \( \alpha \) is a coefficient characterizing the relation between the dissolved oxygen content in ice and water during time \( \Delta t \).

The following formula is used for calculating the reaeration coefficient

\[ k = m_0 \cdot D_e \]

where \( m_0 \) is a parameter determined by trial and error by formula (3) on the basis of the data of on-site observations of the oxygen regime of an operating reservoir.

Since \( K = f(\text{BOD}_0) \) and \( k = f(D_e) \), system of equations (3)-(6) does not have an analytical solution, i.e., is solved numerically, during which the average monthly values of \( K_i, \text{BOD}_0, k_i, \) and \( D_{ki} \) are determined simultaneously by trial and error on the basis of the known average annual values of \( K_0, \text{BOD}_0, k_0, \) and \( D_0 \). Along with this main method of calculating the oxygen regime of reservoirs an approximate analytical method can also be used, according to which the values of \( K_i, \text{BOD}_i, k_i, \) and \( D_{ki} \) are determined successively, but also on the basis of known values of \( K_0, \text{BOD}_0, k_0, \) and \( D_0 \). The main content of these methods, in particular, the method of calculating the reaeration coefficient \( k \), is given below.