1 Introduction

Many neurological disorders originating in the central nervous system (CNS) can be characterised by the electrical activity recorded on the scalp. Such recordings have been catalogued over years of experimentation to form a large body of data for comparative analysis. However, to usefully analyse such recordings taken for an individual, a very practised eye is still required. The disorder for which the analytical procedures are probably most advanced is epilepsy. For the case of epilepsy, scalp EEG can and has been used to approximate the epileptic foci (Bancaud, 1972).

Using depth electrodes, the foci, as well as the intracerebral pathways which are intrained as the seizure develops, have also been characterised. These techniques, while greatly advancing knowledge of the diseased states, are not readily applicable as diagnostic techniques as they require surgical intervention.

The problem of localising intracerebral sources of normal or pathological electrical activity is sufficiently important that many attempts have been made to find appropriate analyses of the scalp EEG which may lead to intracerebral localisation. Most of the studies have been done under the assumption that such signal propagation was comparable with that of an electromagnetic field in homogeneous medium (Shaw and Roth, 1955; Geisler and Gerstein, 1961; Schneider and Gerin, 1970; Henderson et al., 1975). It is now well known that although some small part of the electrical activity of a deep intracerebral nucleus may be electronically conducted to the cortical level, the major part of the scalp EEG is due to the activity of the cortex underlying the scalp recording point (Gastaut et al., 1980) and, to varying degrees, all parts of the brain underlying any particular source. These extraneous signals may or may not be correlated with the signal from the source in question.

In recent years new methods have been developed in the field of spatial processing related to electromagnetic or acoustical antennas in a random field. The fundamental problem solved with these techniques is the identification and the localisation of random sources by a particular processing of the signals received by a limited number of sensors which are frequently corrupted by background noise. These techniques have been shown to be very sensitive and can provide good resolution; general descriptions are given by Picinbono (1966), Widrow et al. (1975), Munier (1975) and Meiroz (1976).

Such methods can only be applied if the following conditions are satisfied:

(a) The source signal to be identified must be uncorrelated with other signals and noise.

(b) The propagation must be linear and locally time-invariant; this means that the signals received by the sensors, in the absence of background noise, are derived from the signals generated by the source, by means of linear filtering.

(c) The entire system consisting of the sources and their surrounding medium must be locally stationary in time; this assumption means that there is a time interval $T$ in which the received signals are well described by a zero-mean stationary stochastic process (Wong, 1971) with a correlation time $t_c$ smaller than $T$; if the filters introduced in $b$ are not strictly time-invariant, we assume that their variations are also small during the same time interval $T$.

These assumptions are necessary conditions for the rigorous application of any signal analysis technique and can never be absolutely verified in practice.

In the present work a method for quantitatively assessing the implications of these basic assumptions has been sought; this should in turn lead to a better estimation of the...
confidence limits for a given analysis procedure. The assumptions a, b and c are verified by using, in particular, the signals which result from the cross-correlation between the implanted electrodes and the cortical electrodes.

These results have permitted a further step, using cortical electrodes only, to separate several strong synchronised patterns of discharges of groups of neurons from local cerebral activity, considered as noise (Lumeau, 1980). At this point only, as the last step of our approach, we can consider the correct localisation of such discharges, using an appropriate propagation model.

2 Principles of the method

2.1 Choice of the model

One way to make such an evaluation is to record simultaneously the assumed input signal \( u_t \) with an electrode placed in the emitting structure, and the received signal \( z_t \) at the cortex. We make the assumption here that the signal, \( u_t \), recorded at the electrode is the source signal. Thus, to test the validity of assumptions a, b and c, we can write:

\[
z_t = y_t + v_t
\]

where \( y_t \) is the output of a linear with input \( u_t \) and \( v_t \) is a noise signal in which no components are correlated with any component of \( u_t \), and \( y_t \) being zero-mean random variables.

The output of a linear causal filter whose impulse response is \( \{ a_k \} \) is given by the convolution:

\[
y_t = \sum_{k=0}^{\infty} a_k u_{t-k}
\]

Because of the noisy nature of \( z_t \), we use the intercorrelation coefficient derived by multiplying the two sides of eqn. 1 by the source signal \( u_{t-1} \) and taking the mathematical expectation:

\[
\Gamma_{zz}(l) = \sum_{k=0}^{\infty} a_k \Gamma_{uu}(l-k) + \Gamma_{uv}(l)
\]

If the noise signal \( v_t \) is not correlated with the source signal \( u_t \), the term \( \Gamma_{uv}(l) \) becomes zero, implying that any part of \( v_t \) correlated with \( u_t \) is a part of \( u_t \). Therefore, when \( \Gamma_{zz}(l) \) is small or zero, \( \Gamma_{zz}(l) \) in eqn. 3 is obtained by a linear filtering of \( \Gamma_{zz}(l) \). It is then possible to examine how well \( \Gamma_{zz}(l) \) is approximated by a linear filtering process of \( \Gamma_{uu}(l) \).

To make this comparison we have used a technique which models the input-output behaviour of a filter independently of its internal structure and known as the autoregressive moving average model (ARMA). The ARMA model has the advantage that it generally allows a good fit with few parameters. It has been developed for use with discrete signals and so is adapted to our needs. In the ARMA model the output for time index \( n \) is given by:

\[
y_n = a_1 y_{n-1} + a_2 y_{n-2} + \ldots + a_p y_{n-p} + b_0 u_n + b_1 u_{n-1} + \ldots + b_q u_{n-q+1}
\]

The unknown parameters \( \{ a_i \} \) and \( \{ b_i \} \) will be called the ARMA parameters. Using the same transformation as in eqn. 3, we deduce from eqn. 4 the following relation between intercorrelations:

\[
\Gamma_{zz}(l) = \Gamma_{uu}(l) = a_1 \Gamma_{zz}(l-1) + \ldots + a_p \Gamma_{zz}(l-p)
\]

\[
+ b_0 \Gamma_{uu}(l) + \ldots + b_q \Gamma_{uu}(l-q+1)
\]

For an arbitrary set of parameters \( \{ \hat{a}_i, \ldots, \hat{a}_p, \hat{b}_0, \ldots, \hat{b}_q \} \), let us denote:

\[
\hat{y}_n = \hat{a}_1 y_{n-1} + \ldots + \hat{a}_p y_{n-p} + \hat{b}_0 u_n + \ldots + \hat{b}_{q-1} u_{n-q+1}
\]

\[
\hat{\Gamma}_{zz}(l) = \hat{a}_1 \hat{\Gamma}_{zz}(l-1) + \ldots + \hat{a}_p \hat{\Gamma}_{zz}(l-p) + \hat{b}_0 \hat{\Gamma}_{uu}(l) + \ldots + \hat{b}_{q-1} \hat{\Gamma}_{uu}(l-q+1)
\]

If a set of parameters \( \{ \hat{a}_i \} \) can be found such that \( y_n - \hat{y}_n \equiv 0 \) or \( \Gamma_{zz}(l) - \hat{\Gamma}_{zz}(l) \equiv 0 \), then the linear relation of eqn. 4 is true for the corresponding values of the parameters, and the hypotheses a and b are tenable. The principal reasons for the nonzero difference are:

(i) imperfect fit of the ARMA model due to a finite number of parameters; the fit can be improved by increasing the number of parameters

(ii) a random error since the intercorrelations are estimated by a finite time average using quantised data.

In fact, the parameters are chosen such that they minimise the squared error:

\[
e^2 = \sum_{l=1}^{\infty} [\Gamma_{zz}(l) - \hat{\Gamma}_{zz}(l)]^2
\]

The algorithm for calculating \( e^2 \) is given in the Appendix. To compare experimental runs, a normalised error or distance is defined as:

\[
D = \left( \sum_{l=1}^{\infty} [\Gamma_{zz}(l) - \hat{\Gamma}_{zz}(l)]^2 \right) / \sum_{l=1}^{\infty} \hat{\Gamma}_{zz}(l)^2
\]

and it will permit us to appreciate the stationarity time if \( D \) can be made to be less than 10 per cent.

2.2 Stationarity time: number of parameters

2.2.1 Determination of the stationarity time. In practice the cerebral medium is constantly changing and we may expect that all parameters are slowly varying in time. Thus, \( a \) priori, stationarity can never be rigorously verified. However, the medium can be said to be quasistationary if the maximum time \( T_q \) during which parameter variation can be neglected, is long with respect to the correlation time \( t_c \) of the signal. We will find that the relation \( t_c < T_q \) is a further necessary condition for the estimation points in more detail, in connection with remarks a and b in Section 2.1.

Let us first suppose that the random errors of the ARMA parameters may be neglected (point b in Section 1). To evaluate the stationarity time \( T_q \), ARMA parameters are estimated by minimising \( D \), given by eqn. 5. Let \( D_1 \) be the corresponding minimum value, computed over the time interval \( \{ t, t + T \} \). Then, with the estimated ARMA parameters, the distance \( D_2 \) is computed over the interval \( \{ t + T, t + 2T \} \). If \( D_2 \) is not significantly greater than \( D_1 \) we expect that all parameters are slowly varying in time. Thus, a priori, stationarity can never be rigorously verified. However, the medium can be said to be quasistationary if the maximum time \( T_q \) during which parameter variation can be neglected, is long with respect to the correlation time \( t_c \) of the signal. We will find that the relation \( t_c < T_q \) is a further necessary condition for the estimation points in more detail, in connection with remarks a and b in Section 2.1.

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*The correlation time \( t_c \), which gives an idea of the minimum time over which two samples of the signal may be considered as uncorrelated, is defined as the equivalent time width of the autocorrelation function, \( B_s \) being the equivalent bandwidth of the spectrum

\[
t_c = \frac{1}{B_s} \approx \frac{1}{\pi \Delta f_{ms}}
\]

for a narrowband signal (Stratanovich, 1967)}