1 Introduction

Tooth mobility has an important significance in the dental clinic. According to clinical findings, dental treatment may be given, and the variations of tooth mobility will be taken into consideration in the formulation of dental prognosis or clinical restoration. Measurements of tooth mobility have already been fully studied and some static (Mühlemann, 1951) and sinusoidal dynamic (Noyes et al., 1968) measurements have been proposed. The forces applied on the teeth, in occlusion or mastication, are of course basically different from static or sinusoidal loads, and should be considered as impulsive loads. In recent dental research there are many reports which deal with the spectra of physical properties caused by impulsive loads applied to the teeth (Manly et al., 1964; D’Hoeijt et al., 1985; Saratani et al., 1984; Hikida et al., 1986). There are, however, very few reports which give full consideration to the relationship between impulsive loads and physical properties based on an analysis of the mechanical elements. It is the purpose of this paper to examine exactly the effects of an impact on the teeth and to propose a new measuring system for evaluating impact response which can be applied to clinical dentistry. The acceleration response obtained with this measuring system is discussed in relation to the elements of the mechanical model.

2 Acceleration response to some excitation

Although a number of models for physical periodontal tissues are feasible, e.g. models containing three or four mechanical elements, a dynamic model proposed by Noyes (Noyes’ model for short) which is composed of two dashpots, two masses and a spring (Noyes and Solt, 1973) is
chosen in this paper. This model can give a close approximation to the mechanical mobility spectrum of periodontal tissues of the maxillary incisor, which can be measured with a sinusoidal swept-audiofrequency method. When the applied excitation force of Noyes' model is \( f(t) \), the displacement is \( x(t) \) and the forces and displacements of the mechanical elements as shown in Fig. 1 are, respectively, as follows:

\[
\begin{align*}
    f(t) &= f_1(t) + f_2(t) \\
    f_1(t) &= c_1 \frac{dx_1}{dt} + kx_1 \\
    f_2(t) &= m_2 \frac{dx_2}{dt} \\
    x(t) &= x_1(t) + x_2(t) + x_3(t) \\
    f_1(t) &= c_2 \frac{dx_2}{dt} \\
    f_2(t) &= m_1 \frac{dx}{dt}^2
\end{align*}
\]  

(1)

If the assumption is that the initial conditions of \( x, x_1, x_2, x_3, \frac{dx}{dt} \) and \( \frac{dx_3}{dt} \) are zero at the time \( t = 0 \), the following equation in \( x(t) \) is obtained:

\[
X(s) = \left(\frac{1}{m_0}\right)\left(\frac{s^2 + Cs + D}{s^2 + As + B}\right)F(s)
\]

(2)

where, \( X(s) \) and \( F(s) \) are the Laplace transforms of \( x(t) \) and \( f(t) \), respectively, and \( A, B, C \) and \( D \) are as follows:

\[
\begin{align*}
    A &= \frac{c_1c_2(m_1 + m_2) + m_1m_2 k}{(c_1 + c_2)m_1m_2} \\
    B &= \frac{c_2 k(m_1 + m_2)}{(c_1 + c_2)m_1m_2} \\
    C &= -\frac{m_2 k + c_1c_2}{(c_1 + c_2)m_2} \\
    D &= \frac{c_2}{(c_1 + c_2)m_2}
\end{align*}
\]

(3)

Next, the input excitation \( f(t) \) applied to the tooth and periodontal tissue is examined in cases of rectangular, triangular and half sine pulse excitation.

2.1 Rectangular pulse excitation

When the input excitation is a rectangular pulse \( f(t) \) with magnitude \( F_0 \), \( f(t) \) is a superposition \( f_1 - f_2 \) of two step functions \( f_1 \) and \( f_2 \). Where \( f_1(t) = F_0 u(t) \), \( f_2(t) = f_1(t - d) \), \( u(t) \) is a unit step function and \( d \) is a weighting time. When two roots of \( s^2 + As + B = 0 \) in eqn. 2 are \( \alpha \) and \( \beta \), \( x(t) \) is derived using the inverse Laplace transform of \( X(S) \).

The acceleration response \( a(t) = \frac{dx(t)}{dt^2} \) is obtained by differentiating the displacement \( x(t) \) twice with respect to \( t \). As the acceleration response becomes a damped vibration in the measured periodontal tissues, two roots, \( \alpha \) and \( \beta \), can be expressed as a pair of complex numbers \( \sigma \pm j\omega \).

\[
\begin{align*}
    a(t) &= a_1(t) - a_2(t) \\
    a_1(t) &= (F_0/m_1)D/B + M_1 e^{-\alpha t} \cos (\omega_1 t - \psi_1)u(t) \\
    a_2(t) &= a_1(t - d)
\end{align*}
\]

(4)

where \( \sigma = A/2 \), \( \omega_1 = \sqrt{B - (A/2)^2} \), \( M_1 = [(B - D)^2/B^2 + (2BC - (A + D))^2/(2B\omega_2^2)]^{1/2} \), \( \psi_1 = \arctan \left(\frac{2BC - (A + D)}{2B\omega_2}\right) \), \( a_1(t) \) and \( a_2(t) \) are step responses to \( f_1 \) and \( f_2 \), respectively.

2.2 Triangular pulse excitation

When the input excitation is a triangular pulse \( f(t) \), \( f(t) \) is a superposition \( f_3 - f_4 - f_2 \) of two ramp functions \( f_3(t) = tF_0 u(t)/d, f_4(t) = f_3(t - d) \) and a step function \( f_2 \) as shown in Fig. 2. When \( f(t) \) in eqn. 1 is a triangular pulse, an acceleration response \( a(t) \) is

\[
\begin{align*}
    a(t) &= a_3(t) - a_4(t) - a_2(t) \\
    a_3(t) &= (F_0/m_1)D/B + M_3 e^{-\alpha t} \cos (\omega_3 t - \psi_3)u(t) \\
    a_4(t) &= a_3(t - d)
\end{align*}
\]

(5)

where

\[
M_3 = \left[\frac{(AD - BC)^2/b^4 + (A(AD - BC))}{2B(B - D)}\right]^{1/2}
\]

\[
\psi_3 = \arctan \left(\frac{\sqrt{A(AD - BC) + 2B(B - D)}}{2\omega_3(AD - BC)}\right)
\]

\( a_2, a_3 \) and \( a_4 \) are responses to \( f_2, f_3 \) and \( f_4 \), respectively.

2.3 Half-sine pulse excitation

When the input excitation is a half-sine pulse \( f(t) \), \( f(t) \) is a superposition \( f_5 - f_6 \) of two sine waves \( f_5(t) = F_0 u(t) \sin \gamma t \) and \( f_6(t) = F_0 u(t - d) \). When \( f(t) \) in eqn. 1 is a half-sine pulse, the acceleration response \( a(t) \) is

\[
\begin{align*}
    a(t) &= a_5(t) - a_6(t) \\
    a_5(t) &= \frac{M_0 F_0/m_1}{M_5} e^{-\alpha t} \cos (\omega_5 t - \psi_5)u(t) + M_6 \sin (\gamma t + \psi_6)u(t) \\
    a_6(t) &= a_5(t - d)
\end{align*}
\]

(6)

where \( \gamma = \pi/d \), \( M_0 = \gamma/\gamma^4 + \gamma^4(A^2 - 2B + B^2) \), \( M_5 = \left[\frac{[(AD - BC) + \gamma(C - A)]^2 + \gamma^2(AD - BC)}{2B^2 + \gamma^2D} - AC(B^2 + \gamma^2) + (A^2 - 2B\gamma^2 + D^2)/(2B\omega_5^2]\right]^{1/2} \), \( \psi_5 = \arctan \left(\frac{\sqrt{A(AD - BC) + 2B(B - D)}}{2\omega_5(AD - BC)} - 2\gamma\omega_5(A - C)\right) \), \( M_6 = (1/\gamma)(\gamma^2(A - C) - (AD - BC))^2 + (\gamma^2B + (AD - BC)) \), \( \psi_6 = \arctan \left(\frac{\gamma^2A(C - A) - (AD - BC)}{\gamma^2B + (AD - BC)}\right) \).

2.4 Simplification of periodontal physical model

Noyes and SolT (1973) reported that the five mechanical elements which are obtained from the mechanical mobility spectrum of maxillary incisors are \( m_1 = 2.9 \) g, \( c_1 = 63 \) N s m\(^{-1}\), \( k = 10^6 \) N m\(^{-1}\), \( c_2 = 980 \) N s m\(^{-1}\) and \( m_2 = 2 \) kg.