1 Introduction

A clinically important and frequently used method of examination of the urinary bladder is ordinary cystometry, i.e. a relative slow filling (20–50 ml min⁻¹) of the bladder by sterile isotonic saline or gas with continuous measurement of the intravesical pressure. If the perivesical pressure is registered simultaneously, the transmural pressure can be calculated and a measure of the elasticity of the bladder wall may then be obtained (Bjerle, 1974).

Expansion of the bladder causes increased tension in the wall but when the stimulus is removed the tension decreases (Coolset et al., 1975). The stress-relaxation curve following a 'step' volume infusion has been shown to be multiexponential with time constants of up to several minutes (Kondo et al., 1972; Kondo and Susset, 1974). As a result, stepwise cystometry has been proposed instead of continuous-filling cystometry. The latter must be performed at an extremely low filling rate to avoid a relaxation phenomenon if elastic properties of the bladder wall are to be evaluated correctly (Coolset et al., 1973).

In this study, we have performed very fast stepwise cystometry in the rat both in vivo and post mortem. The rats were found to be very sensitive to fast volume inputs in vivo, showing spontaneous detrusor contractions shortly after the volume steps. Post mortem, however, the detrusor contractions normally ceased within one hour of death and, therefore, the bladder wall relaxation curves of these measurements were used to evaluate a hydrodynamic model.

We consider that a hydrodynamic model of the bladder is more convenient to use than 'spring-dashpot' models derived from pressure/volume measurements. The modulus of elasticity may be easy to calculate for bladder strips in vitro (Coolset et al., 1976), but far more uncertain to derive from transmural pressure registration of the whole bladder. It requires the assumption of spherical geometry to use Laplace's law (tension = pressure × radius/2) and estimation of the wall thickness to calculate the stress (stress = tension/thickness) to be used in Hooke's law, both assumptions which are questionable.

The aims of this study were:

(a) to describe a hydrodynamic model of the urinary bladder
(b) to perform cystometry with very fast volume steps
(c) to evaluate the optimal mathematical functions corresponding to the stress-relaxation curves.

2 Theory

The mechanical behaviour of strips of urinary bladder wall in vitro has been described by models consisting of a network of Newton (viscosity) and Hooke (elasticity) elements (Fig. 1, van Mastrigt et al., 1978a). The structure and equilibrium equation of a single mechanical Maxwell element is given in Fig. 2 together with a hydrodynamic equivalent consisting of compliance and viscous resistance elements in parallel. Fig. 3 shows the structure and equilibrium equation of a single mechanical Kelvin element together with its hydrodynamic equivalent.

By comparing the mechanical and hydrodynamic elements and equations, elasticity $E$ and viscosity $\eta$ will be transformed into inverted compliance $1/C$ and viscous resistance $R$, respectively, for the hydrodynamic equivalent. We also can see that the $E$ and $\eta$ elements in series
The impedance $Z(s)$ of $R_i$ and $C_i$ elements in parallel is given by

$$Z(s) = \frac{R_i\frac{1}{sC_i}}{R_i + \frac{1}{sR_iC_i}} = \frac{R_i}{1 + sR_iC_i}$$

and the impedance can thus be written

$$Z(s) = \frac{1}{sC_0} + \sum_{i=1}^{n} \frac{R_i}{1 + sR_iC_i}$$

(2)

A unit step volume infusion can be mathematically described as $V(t) = V_n H(t)$, where $H(t)$ is the Heaviside function. The flow $F(t) = (dV(t)/dt) = (d/dt)(V_n H(t)) = V_n \delta(t)$ where $\delta(t)$ is the Dirac function and $\int_0^\infty \delta(t)dt = 1$.

Transformation of flow into the frequency domain is given by the Laplace transform of $F(t)$:

$$F(s) = \mathcal{L}[F(t)] = \mathcal{L}[V_n \delta(t)] = V_n$$

Eqn. 1 can be written as

$$P(s) = F(s)Z(s)$$

(3)

The pressure $P(t)$ after a unit step volume infusion, i.e. an impulse of flow, is determined by the inverse Laplace transform of eqn. 3:

$$P(t) = \mathcal{L}^{-1}[P(s)] = V_n\left\{ \frac{1}{C_0} + \sum_{i=1}^{n} \frac{1}{C_i} \exp (-t/R_iC_i) \right\}$$

(4)

### 3 Materials and methods

Ten Sprague-Dawley rats, weight 535–671 g, mean 614 g, were examined in vivo under neuroleptanalgesia and post mortem after the detrusor instability had ceased, which normally occurred within one hour of death. Post mortem rats were used to evaluate the hydrodynamic models. The abdomen was opened and the bladder wall punctured by two thin needles fixed to plastic catheters. Through one of the catheters, unit volume steps of saline were introduced into the bladder within 0.5 s while pressure was measured with a pressure transducer (Hewlett-Packard 1280 C) connected to the other. Leakage of the system was prevented by ligature of the proximal urethra. The intravesical pressure was assumed to reflect the mechanics of the bladder wall itself, as no contribution from pressure outside the bladder was present because the abdomen was open. The characteristics of the catheter system were measured and showed a flat frequency response up to 5 Hz.

The maximum measured bladder volumes were in the range 1.9–4.4 ml (mean 2.9 ml) and were determined using a final continuous cystometry with a filling rate of 2.4 ml/min⁻¹. The maximum volume was defined as the volume at which the intravesical pressure reached 13.3 kPa (100 mmHg). The pressure rise was then extremely steep and at about 20 kPa the pressure started to plane out or even decrease, probably because of disruption of tissues or leakage of the system. Stepwise cystometry started at about 50 per cent of maximum bladder volume, and subsequently the volume was increased in steps of 0.2–0.5 ml up to 90–95 per cent of the maximum volume.

The stress-relaxation recordings were digitised up to 1 min after the unit steps and fitted to optimum exponential functions using a special program (PROVENCHER and VOGEL, 1980). The program was modified at our laboratory to run on a PDP-11/24, RSX-11M operating