BENDING OF CANTILEVER RECTANGULAR PLATES WITH THE EFFECT OF TRANVERSE SHEAR DEFORMATION

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Abstract

On the basis of Reissner's theory, the exact solutions of the bending of cantilever rectangular plates are obtained by means of the concept of generalized simply-supported boundary. From the results obtained, it can be found that the method is valid.

Key words Reissner's theory, cantilever rectangular plate, symmetrical and asymmetrical bending, generalized simply-supported boundary, exact solution

I. Introduction

As new kinds of material appear and are widely applied, it is necessary to analyse the effect of transverse shear deformation on the bending of a plate. The bending of cantilever rectangular plates has long been regarded as one of the most difficult problems. On the basis of Reissner's theory, this paper is devoted to analysing the symmetrical and unsymmetrical bending of cantilever rectangular plates and the exact solutions are obtained. By using the concept of generalized simply-supported boundary and the superposition principle, the fundamental problem is reduced to solving a system of infinite dimensional linear algebraic equations. Because of employing the trigonometric series, two additional equations must be introduced, which are not the same as those obtained from the corner conditions in the theory of thin plates and result in the mathematics method itself. By solving the system of equations the deflection and bending moment can be obtained and the effects of the thickness of the plate on bending are numerically studied.

II. Mathematics Description and Solutions of the Problem

Assume that the length and breadth as well as thickness of an isotropic cantilever rectangular plate are \(a\), \(b\) and \(h\) and that the plate is subjected to a uniformly transverse load \(q\). And also assume that the edge \(x=0\) of the plate is clamped and the other three edges \(y=0, y=b\) and \(x=a\) are all free
Yang Xiao, Ning Jian-guo and Cheng Chang-jun

In this case, the governing equations are given as:

\[
\Delta^2 w = \frac{q}{D} - \frac{(2-v)h^2}{10(1-v)D} \Delta q \quad \Delta \Phi = \frac{10}{h^2} \Phi = 0
\]

\[
w = \beta_z = \beta_w = 0 \quad \text{for } x = 0
\]

\[
M_z = M_{zz} = V_z = 0 \quad \text{for } x = a
\]

\[
M_y = M_{yy} = V_y = 0 \quad \text{for } y = 0, b
\]

Fig. 1

where \( D = \frac{Eh^3}{12(1-v^2)} \), \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \), and \( E, v \) are the material constants, \( w, \Phi \) are the deflection and stress function, respectively, \( V_z \) and \( V_y \) are transverse shear forces given by

\[
V_z = -D \frac{\partial}{\partial x} \Delta w \quad \frac{(2-v)h^2}{10(1-v)} \frac{\partial q}{\partial x} + \frac{\partial \Phi}{\partial y}
\]

\[
V_y = -D \frac{\partial}{\partial y} \Delta w \quad \frac{(2-v)h^2}{10(1-v)} \frac{\partial q}{\partial y} - \frac{\partial \Phi}{\partial y}
\]

bending twisting moments are

\[
M_z = -D \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + h^2 \frac{\partial V_z}{\partial x} - \frac{\nu h^2}{10(1-v)} q
\]

\[
M_y = -D \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) + h^2 \frac{\partial V_y}{\partial y} - \frac{\nu h^2}{10(1-v)} q
\]

\[
M_{zy} = -D (1-v) \frac{\partial^2 w}{\partial x \partial y} + \frac{h^2}{10} \left( \frac{\partial V_z}{\partial y} + \frac{\partial V_y}{\partial x} \right)
\]

and rotation angles are

\[
\beta_y = -\frac{\partial w}{\partial x} + \frac{h^2}{5(1-v)D} V_z
\]

\[
\beta_y = -\frac{\partial w}{\partial y} + \frac{h^2}{5(1-v)D} V_y
\]

Generally speaking, it is very difficult to solve the boundary value problem (2.1)-(2.3). However, by employing the concept of generalized simply-supported boundary and the principle of superposition, it can solve the boundary value problem. It is clear that the solution of (2.1)-(2.3) may be a proper combination of the solutions of the following problems.

Problem (I) Assume that the rectangular plate is subjected to a uniformly transverse load \( q \) and that all the edges are simply-supported. Hence, the boundary value problem is given as

\[
\Delta^2 w_1 = \frac{q}{D} - \frac{(2-v)h^2}{10(1-v)D} \Delta q, \quad \Delta \Phi = \frac{10}{h^2} \Phi_1 = 0
\]

\[
w_1 = M_{1z} = \beta_{1z} = 0 \quad \text{for } x = 0, a
\]

\[
w_1 = M_{1y} = \beta_{1y} = 0 \quad \text{for } y = 0, b
\]