Summary. — Electron crystallization due to sufficiently strong Coulomb repulsions leads to a model of electrons oscillating about lattice sites in harmonic potentials. We have studied the orbital diamagnetism of such a localized Wigner oscillator in an applied magnetic field of arbitrary strength. The two contributions from orbital angular momentum and from Langevin-Pauli behaviour are separately calculated using Feynman's theorem. Possible relevance to the melting curve of the Wigner crystals in a magnetic field is pointed out.

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1. — Introduction.

Following Wigner's pioneering work (1,2), a good deal of interest has been focused on electron crystallization due to sufficiently strong Coulomb repulsions. Genuine three-dimensional Wigner crystals, however, have still not been unambiguously identified though a lot of work on highly compensated semiconductors in strong magnetic fields points to the relevance of Wigner crystallization in this area (3). Of course, extensive evidence for two-dimensional

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electronic ordering on the surface of liquid helium has been presented (4).

As to the observation of three-dimensional crystals, the melting temperature of the Wigner crystal is clearly highly relevant and approximate theories have been proposed (5,6). However, in the compensated semiconductors, the question arises as to the effect of strong magnetic fields on the melting curve: this was the initial motivation underlying the present study. This is because of the appropriate Clausius-Clapeyron-type equation

\[
\frac{dT}{d\mathcal{H}} = \frac{\Delta M}{\Delta S},
\]

which would on integration yield the melting temperature \( T_m \) say as a function of magnetic field \( \mathcal{H} \), given the knowledge of the change in magnetization \( M \) and entropy \( S \) across the transition.

Here we focus mainly on one aspect of the problem, namely the contribution to \( \Delta M \) of orbital diamagnetism in the Wigner crystal phase. As is well known (3), in a jellium model the restoring force pulling a displaced electron back to its own lattice site is determined by the uniform background density \( \varrho \) of positive charge through

\[
F = -k \varrho, \quad k = \frac{2\pi \varrho e^2}{3}.
\]

Naturally, in say a classical version of jellium, we are always dealing with a strong-coupling regime in which the plasma parameter \( \Gamma \) defined by

\[
\Gamma = \frac{e^2}{ak_B T}, \quad \varrho = \frac{3}{4\pi a^3}
\]

is greater than the critical value of about 180 required to induce crystallization; the same qualification is true for the quantal Wigner crystal, i.e. one must work at a density low enough for the solid phase to be stable.

Thus the problem we address in the following section is that of calculating the orbital diamagnetism, thermally averaged of a single localized Wigner oscillator. While we recognize that full inclusion of coherence effects associated with the Wigner crystal poses a much more difficult problem, contact with this can be made by regarding our present treatment as an Einstein-like model. The oscillator has a natural frequency, \( \omega_0 \) say, given by \( \omega_0^2 = k/m \), which can be likened to a plasma frequency or longitudinal optic mode having only weak dispersion in three dimensions.

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