MATRIX PERTURBATION METHOD FOR THE VIBRATION PROBLEM OF STRUCTURES WITH INTERVAL PARAMETERS*

Qiu Zhi-ping (~) Chen Su-huan (~) Liu Zhong-sheng (~)
(The Centre of Computational Mechanics Jilin University of Technology, Changchun)
(Received Dec. 3, 1993; Communicated by Tang Li-min)**

Abstract

When the parameters of the structures are uncertain, the structural natural frequencies become uncertain. In this paper, we deal with the vibration problem of the structure with interval parameters, the eigenvalue problem of the structures with interval parameters is transformed into two different eigenvalue problems to be solved. The perturbation method is applied to the vibration problem of the structures with interval parameters, the numerical results show that the proposed method is sufficiently accurate and needs little computational efforts.

Key words natural frequency, interval parameter, matrix perturbation

I. Introduction

The vibration theory for structures with deterministic parameters has been well developed. However, it is often impossible to describe the structural parameters with deterministic version because a large amount of uncertain information exists in practical engineering, for example, manufacture errors, errors in observation. Such uncertainty requires that structural parameters must be described in uncertain version. There exist three methods to describe this uncertainty in structural parameters: probability description, fuzzy sets description and interval parameters description, i.e. unknown-but-bounded parameters description. The vibration problem with random parameters (probability description and fuzzy parameters) has been discussed. It is noted that a few research work exists on the vibration problem of the structures with interval parameters.

When the uncertain parameters are described using interval version, the associated natural frequencies will be interval. This paper discusses how to compute the bounds of eigenvalues of structural vibration systems with interval parameters, an efficient method to be discussed in this paper is the perturbation method for constructing the upper and lower bounds of eigenvalues for structures with interval parameters. This technique has the advantage of computing interval eigenvalues without interval arithmetic.

*Project supported by the National Natural Science Foundation of China
**First received Nov. 30, 1992
II. Generalized Interval Eigenvalue Problem

It is often desirable in various ways of application to obtain solutions to the equation $Ku = \lambda Mu$, in which $K$ and $M$ are affected by uncertainties. One becomes therefore concerned with determining the tolerance in each component $\lambda_i$ of the solution, knowing the tolerance inherent in elements $k_{ij}$ and $m_{ij}$, $i, j = 1, 2, \ldots, n$. Such a problem pertains usually to dynamic model data are gathered from field or experimental observations which certainly lack precision.

Let us consider eigenvalue problem for the structural vibration

$$Ku = \lambda Mu$$ (2.1)

subject to

$$K < K < \bar{K} \quad \text{or} \quad k_{ij} < k_{ij} < \bar{k}_{ij} \quad (i, j = 1, 2, \ldots, n)$$ (2.2)

$$M < M < \bar{M} \quad \text{or} \quad m_{ij} < m_{ij} < \bar{m}_{ij} \quad (i, j = 1, 2, \ldots, n)$$ (2.3)

where $K = (k_{ij})$ is the stiffness matrix, $M = (m_{ij})$ is the mass matrix, $u$ is the mode shape and $\lambda$ is the square of the frequency of free vibration. Both $K$ and $M$ are symmetric, and $K$ is positive semi-definite and $M$ is positive definite, $K = (k_{ij})$ and $\bar{K} = (\bar{k}_{ij})$ are the minimum and maximum allowable stiffness matrices of the system, but $K$ is uncertain and it ranges over Eq. (2.2), $M = (m_{ij})$ and $\bar{M} = (\bar{m}_{ij})$ are the minimum and maximum allowable mass matrices of the system, but $M$ is uncertain and it ranges over Eq. (2.3).

With the interval matrix notation, Eqs.(2.2) and (2.3) can be written as

$$K \in K', \quad M \in M'$$ (2.4)

in which $K' = [K, \bar{K}]$ is a positive semi-definite interval matrix (1) and $M' = [M, \bar{M}]$ is a positive definite interval matrix (2).

For the sake of simplicity, by means of Eq. (2.4), Eqs.(2.1), (2.2) and (2.3) can be expressed

$$K'u = \lambda'M'u.$$ (2.5)

Eq. (2.5) is called a generalized interval eigenvalue problem (3).

The basic problem is: the given central matrices $K = (K + \bar{K})/2$ and $M = (M + \bar{M})/2$ of $K'$ and $M'$, and the uncertain matrices $\Delta K = (\bar{K} - K)/2$ and $\Delta M = (\bar{M} - M)/2$ of $K'$ and $M'$, how to find interval eigenvalues $\lambda'$, which is the smallest width and encloses all possible eigenvalues $\lambda$ satisfies $Ku = \lambda Mu$, when $K$ and $M$ assume all possible combinations. In other words, we seek a hull, i.e.

$$\lambda' = (\lambda_i') = [\tilde{\lambda}_i, \bar{\lambda}_i], \quad \lambda_i = [\tilde{\lambda}_i, \bar{\lambda}_i] \quad (i = 1, 2, \ldots, n)$$ (2.6)

to the set

$$\Gamma = \{ \lambda: \lambda \in R, \quad K'u = \lambda'M'u, \quad u \neq 0, \quad K \in K', \quad M \in M' \}$$ (2.7)

where

$$\lambda_i = \min_{K \in K', \quad M \in M'} \lambda_i(\langle K, M \rangle)$$ (2.8)

$$\bar{\lambda}_i = \max_{K \in K', \quad M \in M'} \lambda_i(\langle K, M \rangle)$$ (2.9)

in which (9,10)}