THE NONLINEAR NUMERICAL ANALYSIS METHOD FOR FRAMES

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Abstract

This paper gives the direct formulas of stiffness matrixes of two kinds of Kirchhoff nonlinear elements under total-Lagrange coordinate. For the first one, it includes not only the quadric terms of increments of strain and displacement but also the influence of rotations. For the second one, it is simplified and its nonlinear is considered by taking into account the influence of axial force on the equilibrium equation in the linear beam theory.

The nonlinear equation obtained from both of the above-said elements is solved by mixed Newton-Raphson method, and by comparing the results obtained from two kinds of nonlinear beam some important conclusions that we can know how to use them right are given in our paper.

I. Introduction

With the development of various synthetic materials higher in strength, lighter in weight, and better in elasticity, more and more flexible bars are used in engineering, and the classic small deformation theory has not fulfilled requirement of modern structure analyses. The importance of nonlinear analysis is rising day by day. In recent years the nonlinear finite element analysing frames have been studied by many researchers[1-3]. More generally, the four-or eight-nodal point isoparametric quadrilateral finite element, or tow-dimensional curved beam element is adopted in calculating plane frames, so that it is necessary that more elements are selected in one bar in order to describe large deformation, and it leads to disadvantages that it takes more time to compute and it is more complex to present formulation.

In this paper, we give the direct formulations of stiffness matrixes of two kinds of Kirchhoff nonlinear beam elements, which are of advantages of simple formulation, apparent geometrical physic significance, and more convenient in transforming coordinate. Numerical examples in this paper show that the behavior and the accuracy of the general nonlinear beam element for geometricall nonlinear applications are still good as long as its element is selected small enough. The simplified nonlinear beam element can be applied only in those weak nonlinear frames in which the influence of axial force is larger than that of the others.

II. General Nonlinear Beam Element

1. Element geometry

Fig. 1 presents a constant section across beam-element \( ij \) under assumption of Kirchhoff. For such an element the node displacements and forces can be written as
Considering the coupling between the axial deformation and the lateral one during the large deformation process, we take element strain as
\[ \{ \epsilon \} = \{ \epsilon_0 \} + \{ \epsilon_L \} \]  
(2.2)

where linear strain
\[ \{ \epsilon_0 \} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 w}{\partial x \partial \theta} \\ \frac{\partial v}{\partial x} & \frac{\partial^2 v}{\partial x^2} & \frac{\partial^2 w}{\partial x \partial \theta} \\ \frac{\partial \theta}{\partial x} & \frac{\partial^2 \theta}{\partial x^2} & \frac{\partial^2 w}{\partial x \partial \theta} \end{bmatrix}^T \]  
(2.3)

and nonlinear strain
\[ \{ \epsilon_L \} = \frac{1}{2} \left\{ \begin{array}{c} \left( \frac{\partial u}{\partial x} \right)^2 \\ \left( \frac{\partial v}{\partial x} \right)^2 \end{array} \right\} \]

Further, assuming displacement field
\[ \{ D \} = \{ u \ v \ w \} = [N]\{ q \} \]

Correspondingly, the shape function is
\[ [N] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 & 0 & N_4 & 0 & N_6 & 0 & 0 & 0 & N_6 \\ 0 & 0 & N_3 & 0 & -N_4 & 0 & 0 & 0 & N_6 & 0 & -N_6 & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 \end{bmatrix} \]  
(2.4)

in which
\[ N_1 = 1 - x/L, \quad N_2 = x/L, \quad N_3 = 1 - \frac{3x^2}{L^2}, \quad N_4 = \frac{3x^2}{L^2}, \quad N_5 = -\frac{x^2}{L^2}, \quad N_6 = \frac{x^2}{L^2} \]

The strains can be written as
\[ \{ \epsilon \} = [\bar{B}]\{ q \} = ([\bar{B}_0] + [\bar{B}_L])\{ q \} \]

then
\[ [\bar{B}_0] = \begin{bmatrix} N_1 \n N_1 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 & 0 & N_4 & 0 & N_6 & 0 & 0 & 0 & N_6 \\ 0 & 0 & N_3 & 0 & -N_4 & 0 & 0 & 0 & N_6 & 0 & -N_6 & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 \end{bmatrix} \]  
(2.5)

and
\[ [\bar{B}_L] = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

in which