THE INTEGRAL AS A FUNCTION OF THE UPPER LIMIT AND DEPENDING ON A PARAMETER TO SOLVE DRAWING THROUGH IDLING ROLLS

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Abstract

The velocity and strain-rate fields which are different from Avitzur’s have been established in Cartesian coordinates. Using the integral as a function of the upper limit and integration depending on a parameter, an analytical upper-bound solution to drawing stress through idling rolls has been obtained in this paper.

Key words  drawing through idling rolls, integral as a function of the upper limit, integration depending on a parameter, analytical solution

1. Introduction

A kinematically admissible continuous velocity field in cylindrical coordinates is first proposed by B. Avitzur to solve the drawing through idling rolls and an upper-bound solution is as follows

\[
\frac{\sigma_{sl}}{(2/\sqrt{3})\sigma_s} = \frac{\sigma_{sb}}{(2/\sqrt{3})\sigma_s} + \ln\left(\frac{h_0}{h_1}\right) + \frac{1}{4\pi} \left(\frac{h_0}{R}\right) \left(1\frac{h_0}{h_1}\right) - 1
\]

\[
+ m \frac{v}{v_1} \left(1 + \frac{R}{h_1} \alpha^2\right) \sqrt{\frac{h_1}{R}}
\]

\[
\left(2\tan^{-1}\sqrt{\frac{R}{h_1}} \alpha - \tan^{-1}\sqrt{\frac{h_0}{h_1} - 1}\right)
\]

\[
+ \left(\sqrt{\frac{h_1}{R}} \sqrt{\frac{h_0}{h_1} - 1} - 2\alpha\right)
\]

(a)

where \(\sigma_{sl}\) is stress of drawing through idling rolls, \(\alpha\) is no slip angle. For calculation, one can take \(\alpha = \theta/2\) (\(\theta\) is maximal contact angle). O. Hoffman, G. Sacs studied the problem previously by slab method\(^2\). However, the emphasis of this paper is on using the integral as a function of the upper limit, integrating depending on a parameter, and establishing continuous velocity field in Cartesian coordinates under Karman’s assumption to get an analytical solution for drawing through idling rolls.
II. Assumed Velocity Fields

When rolls are powered only by front tension of the strip, the strip rolling is known as the drawing through idling rolls. The forming region of this kind of drawing without back tension is shown in Fig. 1. Set the thickness at entry is $h_0$ and at the exit the thickness is $h_1$, drawing tension stress and velocity are $\sigma_{st}$ and $v_1$. From Karman's assumption, that is, the $x, y, z$ directions are directions of principal stress; stress $\sigma_z$ and velocity $v_z$ are uniformly distributed over cross sections: At the distance $x$ from entry, the thickness is $h_x$ then

$$h_x = 2R + h_1 - 2\sqrt{R^2 - x^2} \quad (2.1)$$

$$h_x = 2R + h_1 - 2R\cos\alpha \quad (2.2)$$

$$\sin\alpha = \frac{x}{R}$$

(2.1) and (2.2) are respectively Cartesian coordinate equation and parametric equations of the contact arc. Notice that flow per second is constancy, then

$$h_x v_x = h_0 v_0 = h_1 v_1 = h_x v_x = v\cos\alpha, \quad \frac{V}{BT} = c \quad (2.3)$$

where $c$ is flow constant, it is equal to total drawn volume divided by the drawing time $T$ and width $B$. Substituting (2.1) and (2.2) into (2.3) leads to

$$v_x = \frac{\sigma}{2R + h_1 - 2\sqrt{R^2 - x^2}}$$

or

$$v_x = \frac{\sigma}{2R + h_1 - 2R\cos\alpha} \quad (2.4)$$

In formula (2.4), $c$ is a constant in deforming zone which is independent from $x$ and $\alpha$.

From Cauchy equation and take note of (2.4)