THE ASYMPTOTIC STABILITY OF THE LINEAR, DISCRETE
LARGE-SCALE SYSTEMS

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Abstract

In this paper, we directly use the linear norm Liapunov function to investigate the
stability of the linear discrete large-scale systems and obtain some criteria for the
asymptotic stability of such a system.

Key words discrete large-scale systems, stability, Liapunov function

I. The Linear Discrete Large-Scale System with Constant Coefficients

Lemma G is a matrix, all of whose elements are positive.

If $X(k)$ and $Y(k)$ are the solutions of

\[
\begin{align*}
X(k+1) &= GX(k), \\
Y(k+1) &= GY(k), \\
X(k_0) &= X_0, \\
Y(k_0) &= Y_0
\end{align*}
\]

respectively and $X_0 = Y_0$, then $X(k) \leq Y(k)$ is true for all $k \in I = \{0, 1, 2, \ldots\}$.

Consider the linear discrete large-scale systems with constant coefficients

\[
x_i(k+1) = \sum_{j=1}^{n} a_{ij} x_j(k) \quad (i = 1, \ldots, n)
\]

that is

\[
X(k+1) = AX(k)
\]

The system (1.1) is divided into the isolated subsystems

\[
X_{nr}(k+1) = A_{nr} X_{nr}(k) \quad (r = 1, \ldots, m_s, \quad n_1 + \cdots + n_{r-1} + n_r = n)
\]

\[
X_{nr}(k) = \begin{pmatrix}
x_{n_1 + \cdots + n_r - 1 + 1}(k) \\
\vdots \\
x_{n_1 + \cdots + n_r}(k)
\end{pmatrix}
\]

\[
A_{nr} = \begin{pmatrix}
a_{n_1 + \cdots + n_r - 1 + 1} & a_{n_1 + \cdots + n_r - 1 + 1} & \cdots & a_{n_1 + \cdots + n_r - 1 + 1} \\
a_{n_1 + \cdots + n_r} & a_{n_1 + \cdots + n_r} & \cdots & a_{n_1 + \cdots + n_r}
\end{pmatrix}
\]

Theorem 1.1 The equilibrium of the system (1.1) is asymptotically stable if the
successive principal minor determinants of the matrix $D = (d_{ij})$,
are all positive.

Proof Consider Liapunov function for the subsystems (1.2) defined by

\[ V_r(x, k) = \sum_{i=1}^{n_r} a_i |x_i| \quad (r = 1, \ldots, m) \]

where \( a^T = (a_{n_{r-1}+1}, \ldots, a_{n_r}) > 0 \) is an arbitrary constant vector. Then

\[ \Delta V_{r(1,1)}(x, k) = \sum_{i=1}^{n_r} a_i \left[ \sum_{j=1}^{n_r} a_{ij} x_j(k) \right] - \sum_{i=1}^{n_r} a_{ij} |x_i(k)| \]

\[ \leq \sum_{i=1}^{n_r} a_i \left\{ \left( |a_{ij}| - 1 \right) |x_i(k)| + \sum_{j=1}^{n_r} |a_{ij}| |x_j(k)| \right\} \]

(r = 1, \ldots, m).

We take Liapunov function for the system (1.1) defined by

\[ V(x, k) = \sum_{r=1}^{m} V_r(x, k). \]

It is evident that the function \( V \) is a positively definite matrix and we have

\[ \Delta V_{(1,1)}(x, k) = \sum_{r=1}^{m} \Delta V_{r(1,1)}(x, k) \]

\[ \leq - \sum_{r=1}^{m} \sum_{i=1}^{n_r} a_i \left\{ \left( 1 - |a_{ij}| \right) |x_i(k)| - \sum_{j=1}^{n_r} |a_{ij}| |x_j(k)| \right\} \]

\[ = -a^T D W \triangle + B^T W \]

where \( W = [x_1, \ldots, x_n] \), \( a^T D = B^T \).

By hypothesis we know that \( D \) is M-Matrix, thus there is \( D^{-1} \succeq 0 \), and so \( a = (D^{-1})^T B \), and since the diagonal elements are all positive, we may choose \( B \) such that \( B \succ 0 \) then \( a \succ 0 \). Thus \( \Delta V_{(1,1)}(x, k) \) is negative definite for all \( x \in \mathbb{R}^n \) and \( k \in I \). This completes the proof.

In particular, we take a second-order system as an example for illustrating

\[ \begin{cases} x_1(k+1) = a_{11} x_1(k) + a_{12} x_2(k) \\ x_2(k+1) = a_{21} x_1(k) + a_{22} x_2(k) \end{cases} \]

(1.3)

Consider the isolated subsystem

\[ \begin{cases} x_1(k+1) = a_{11} x_1(k), \\ x_2(k+1) = a_{22} x_2(k). \end{cases} \]

By theorem 1.1 the sufficient condition that the equilibrium of the system (1.3) is asymptotically stable is true as follows: