THE ASYMPTOTIC STABILITY OF THE LINEAR, DISCRETE LARGE-SCALE SYSTEMS

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Abstract
In this paper, we directly use the linear norm Liapunov function to investigate the stability of the linear discrete large-scale systems and obtain some criteria for the asymptotic stability of such a system.

Key words discrete large-scale systems, stability, Liapunov function

I. The Linear Discrete Large-Scale System with Constant Coefficients

Lemma  G is a matrix, all of whose elements are positive.

If \( X(k) \) and \( Y(k) \) are the solutions of

\[
\begin{align*}
X(k+1) &= GX(k), \\
X(k_0) &= X_0
\end{align*}
\]

respectively and \( X_0 = Y_0 \), then \( X(k) \leq Y(k) \) is true for all \( k \in I = \{0, 1, 2, \ldots\} \).

Consider the linear discrete large-scale systems with constant coefficients

\[
x_i(k+1) = \sum_{j=1}^{n} a_{ij} x_j(k) \quad (i=1, \ldots, n)
\]

that is

\[
X(k+1) = AX(k)
\]

The system (1.1) is divided into the isolated subsystems

\[
X_{n_r}(k+1) = A_{n_r} X_{n_r}(k) \quad (r = 1, \ldots, m, n_1 + \ldots + n_{r-1} + n_r - n)
\]

\[
A_{n_r} = \begin{pmatrix}
\alpha_{n_1 + \ldots + n_{r-1} + 1} & \alpha_{n_1 + \ldots + n_{r-1} + n_r} & \ldots & \alpha_{n_1 + \ldots + n_{r-1} + n_r + 1} \\
\vdots & \ddots & \ddots & \vdots \\
\alpha_{n_1 + \ldots + n_{r-1} + n_r} & \alpha_{n_1 + \ldots + n_{r-1} + n_r + 1} & \ldots & \alpha_{n_1 + \ldots + n_{r-1} + n_r + n_r}
\end{pmatrix}
\]

Theorem 1.1 The equilibrium of the system (1.1) is asymptotically stable if the successive principal minor determinants of the matrix \( D = (d_{ij})_1 \).
\[
d_{ij} = \begin{cases} 
1 - |a_{ij}| & (i = j), \\
- |a_{ij}| & (i \neq j)
\end{cases}
\]

are all positive.

**Proof** Consider Liapunov function for the subsystems (1.2) defined by

\[
v_r(x,k) = \sum_{i=1}^{n_r} a_i |x_i| (r = 1, \ldots, m)
\]

where \( a^T = (a_{n_r+1}, \ldots, a_{n_r}) > 0 \) is an arbitrary constant vector. Then

\[
\Delta v_{r(1,1)}(x,k) = \sum_{i=1}^{n_r} a_i \left| \sum_{j=1}^{n_r} a_{ij} x_j(k) \right| - \sum_{i=1}^{n_r} a_i |x_i(k)|
\]

\[
\leq \sum_{a_i = n_r+1}^{n_r} a_i \left\{ (|a_{ii}| - 1) |x_i(k)| + \sum_{j=1}^{n_r} |a_{ij}| |x_j(k)| \right\} 
\]

\[
(\text{r}=1, \ldots, m).
\]

We take Liapunov function for the system (1.1) defined by

\[
V(x,k) = \sum_{r=1}^{m} v_r(x,k).
\]

It is evident that the function \( V \) is a positively definite matrix and we have

\[
\Delta V_{r(1,1)}(x,k) = \sum_{q=1}^{m} \Delta v_{r(1,1)}(x,k)
\]

\[
\leq - \sum_{r=1}^{m} \sum_{i=n_r+1}^{n_r} a_i \left\{ (1 - |a_{ii}|) |x_i(k)| + \sum_{j=1}^{n_r} |a_{ij}| |x_j(k)| \right\}
\]

\[
= -a^T D W \triangleq B^T W
\]

where \( W = [|x_1|, \ldots, |x_n|] \), \( a^T D = B^T \).

By hypothesis we know that \( D \) is \( M \)-Matrix. Thus there is \( D^{-1} \geq 0 \), and so \( a - (D^{-1})^T B \), and since the diagonal elements are all positive, we may choose \( B \) such that \( B > 0 \) then \( a > 0 \). Thus \( \Delta V_{r(1,1)}(x,k) \) is negative definite for all \( x \in \mathbb{R}^n \) and \( k \in I \). This completes the proof.

In particular, we take a second-order system as an example for illustrating (1.1)

\[
\begin{cases}
x_1(k+1) = a_{11} x_1(k) + a_{12} x_2(k) \\
x_2(k+1) = a_{21} x_1(k) + a_{22} x_2(k)
\end{cases}
\]

(1.3)

Consider the isolated subsystem

\[
\begin{cases}
x_i(k+1) = a_{ii} x_i(k), \\
x_i(k+1) = a_{ii} x_i(k)
\end{cases}
\]

By theorem 1.1 the sufficient condition that the equilibrium of the system (1.3) is asymptotically stable is true as follows: