THE IMPACT TORSIONAL BUCKLING OF ELASTIC CYLINDRICAL SHELLS WITH ARBITRARY FORM IMPERFECTION

Wang De-yu (王德禹) Zhang Shan-yuan (张善元) Yang Gui-tong (杨桂通)

(Institute of Applied Mechanics, Taiyuan University of Technology, Taiyuan)

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Abstract

A perturbation analysis for the impact torsional buckling of imperfective elastic cylindrical shells subjected to a step torque is given. The imperfection is supposed to be small and has arbitrary form. It is shown that only the imperfection which has the shape of static torsional buckling mode could influence the critical step torque. Finally a formula is presented for the critical step torque.

Key words impact torsional buckling, elastic cylindrical shell, perturbation analysis, initial imperfection sensitivity

I. Introduction

It is well-known that imperfection could decrease the critical buckling load for the structures which are sensitive to imperfection. In the last few decades, a lot of work has been done for the static buckling and dynamic buckling research. The buckling of cylindrical shell, which is known as a typical engineering component and typical imperfection sensitive structure, has got much attention, and the recent review for this problem is in [1].

Generally speaking, the imperfection form is assumed to be the same as the buckling mode, such as the studies for the torsional buckling of cylindrical shell [2, 3]. But in fact, the shape of imperfection is always arbitrary, so it is more important to study the buckling of shells with arbitrary form imperfection. Using a perturbation method, Lockhart [4, 5] studied the impact buckling for the external step pressure loaded cylindrical shell with arbitrary imperfection, and they pointed out that what effect the decrease of critical step pressure is only the initial imperfection which has the shape of classical buckling mode. Lockhart's works show that in the buckling analysis of cylindrical subjected to external uniform pressure, the form of imperfection can be taken the same as that of the buckling mode.

The purpose of this paper is to study the torsional buckling of clamped elastic cylindrical shell with arbitrary imperfection, when it is loaded with a step torque.

II. Dynamic Governing Equation

Consider an elastic cylindrical shell with radius \( R \), thickness \( h \), length \( L \) and density \( \rho \) and loaded with a step torque \( M \), using the Donnell nonlinear dynamic equation, we have:
\[\rho W,_{tt} + D \nabla^4 W + F,_{XX/R} = S(W + \bar{W}, F) \quad (2.1)\]
\[
\frac{1}{Eh} \nabla^4 F - \frac{1}{R} W,_{XX} = -S \left(W, \frac{1}{\epsilon} W, W\right) \quad (2.2)
\]

where
\[
\nabla^4 W = \frac{\partial^4 W}{\partial X^4} + \frac{\partial^4 W}{\partial Y^4}, \quad F,_{XX} = \frac{\partial^2 F}{\partial X^2}
\]
\[
S(P, Q) = \frac{\partial^2 P}{\partial X \partial Y} + \frac{\partial^2 Q}{\partial X \partial Y} - 2 \frac{\partial^2 P}{\partial X \partial Y} - \frac{\partial^2 Q}{\partial X \partial Y}
\]

\(E\) is elastic modulus, \(\nu\) is Poisson's ratio, \(D = Eh^4/12(1 - \nu^2)\), \(F(X, Y)\) is the stress function, \(W\) and \(\bar{W}\) are normal deflection and the initial geometrical imperfection respectively, and they are not required to have the same shape. \(X, Y\) and \(T\) are the axial, circumferential and time co-ordinates. When the two ends of the shell are clamped, the boundary condition are:
\[
W,_{X} = 0 \quad (X = 0 \text{ or } L) \quad (2.3)
\]
\[
- \int_{0}^{\pi R} RF,_{Y} dY = M, \quad \int_{0}^{\pi R} F,_{Y} dY = 0 \quad (X = 0 \text{ or } L) \quad (2.4)
\]
The initial condition is
\[
W = W,_{T} = 0 \quad (T = 0) \quad (2.5)
\]
The following dimensionless quantities are introduced:
\[
x = \pi X/L, \quad y = Y/R, \quad e\bar{W} = \bar{W}/h, \quad w = W/h, \quad t = T\pi^2(D/\rho)^{1/2}/L^2
\]
\[
H = h/R, \quad A^2 = \frac{L^4 E}{\pi^4 DR^4} = \frac{12(1 - \nu^2)L^4}{\pi^4 h^4 R}, \quad \lambda = -\frac{ML^3}{DR^2}, \quad \xi = \left(\frac{L}{\pi R}\right)^2
\]

Let stress function \(F(x, y)\) has the following form:
\[
F(x, y) = -\frac{ML}{2R\pi x} xy + \frac{L^2 E h}{\pi^4} f(x, y) \quad (2.6)
\]

where \(f(x, y)\) is the shape of the initial imperfection, and \(e\) is its amplitude. Thus the simplified basic equations are
\[
\bar{\nabla}^4 f - H w,_{XX} = -H^2 S(w, e\bar{W} + 0.5w) \quad (2.7)
\]
\[
w,_{tt} + \bar{\nabla}^4 w + A^2 f,_{XX} = H A^2 S(w + e\bar{W}, f) - \lambda (w + e\bar{W}),_{XX} \quad (2.8)
\]
\[
w = w,_{XX} = 0 \quad (x = 0 \text{ or } \pi) \quad (2.9)
\]
\[
\int_{0}^{\pi R} f,_{Y} dY = \int_{0}^{\pi R} f,_{XX} dY = 0 \quad (x = 0 \text{ or } \pi) \quad (2.10)
\]
\[
w = w,_{T} = 0 \quad (t = 0) \quad (2.11)
\]

where
\[
\bar{\nabla}^4 = \frac{\partial^2}{\partial x^2} + \xi \frac{\partial^2}{\partial y^2}
\]