THE MATHEMATICAL MODEL OF BUBBLE GROUP NOISE

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Abstract

In this paper the mathematical model of bubble group noise are introduced under the arbitrary conditions by using the method of Euler. The calculation indicates that the simulation results consist with the measured value.

Key words cavitation, cavitation noise, mathematical model

I. Noise at Collapse of an Isolated Bubble

The noise only has comparative big value at collapse period. By Ref. [1], we have:

The radial velocity of bubble-wall of bubble at collapse

$$R = \frac{dR}{dt} = \left\{ -\frac{2}{5\rho} \left( p_{\infty} - p_{w} \right) \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{2P_1}{3(1-\gamma)\rho} \left( \frac{R_0}{R} \right) \right\} \frac{1}{2} - \left( \frac{R_0}{R} \right)^{3\gamma} + \frac{2\sigma}{\rho R} \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right]^{\frac{3}{2}} \right\}$$

(1.1)

The sound pressure amplitude at the point Q with the radial distance r produced by an isolated bubble at collapse

$$p(R, r) = \frac{\sigma}{r} \left[ \left( \frac{R_0}{R} \right)^2 - 3 \right] + \frac{(p_\infty - p_\omega)}{r} \left\{ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{R_0^3}{R^3} \right\}$$

$$- \frac{P_1}{(1-\gamma)r} \left\{ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - \left( \frac{R_0}{R} \right)^{3\gamma} \right] - \frac{R_0^3}{R^3} + \frac{\gamma R_0^{3\gamma}}{R^{(3\gamma-1)}} \right\}$$

(1.2)

where $R$—radius of bubble at collapse; $p_\infty$—pressure at infinity; $p_\omega = p_\omega(T)$—vapour pressure in bubble; $\sigma = \sigma(T)$—surface tension of liquid; $\gamma$—gas constant (adiabatic) of air; $P_1$—gas pressure in bubble at initial moment of collapse; $R_0$—initial radius of bubble at collapse; $T$—temperature of liquid; $\rho$—density of liquid; $t$—time.

Substituting (1.1) into (1.2), we obtain the sound pressure amplitude of an isolated bubble at collapse $p(t, r)$ (See Fig. 1).

Fig. 1 The sound pressure amplitude of isolated bubble at collapse
II. The Mathematical Model of Bubble Group Noise

In practice, the bubbles always appear in the form of the bubble group at the surface of body. The collapse time $0 - T_0$ correspond to the distance $0 - x_0$ on the surface of body for the any given body (See Fig. 2), where $0$ is the collapse origin of bubble. $x_0$ is the collapse terminal point of bubble ($x_0$ correspond to the distance of bubble remove in $T_0$).

![Fig. 2 The distribution of bubbles and its sound pressure](image)

In general, the form of body, velocity and pressure at infinity are all permanent, therefore the flow field can be considered to be steady flow.

Suppose the number of bubbles produced on surface of body per unit time equals $N$ (it is the number of bubble entering into the region of collapse per unit time). Under the steady conditions, $N$ is a constant, its distribution along the $x$ is uniform and the curve of $p(x)$ is steady. Thus, the whole bubble group are steady uniform distribution from $0$ to $x_0$. The number of bubbles in the $\Delta x$ at the $x$ is $N \Delta t = N/V \cdot \Delta x$ (where $V$ is the velocity of liquid at $x$). The sound pressure amplitude at $Q$ produced by any bubble in $\Delta x$ is the sound pressure amplitude corresponding to $x$.

At any moment, the sound pressure amplitude at $Q$ produced by bubble group is a constant that equals a sum of the sound pressure amplitude produced by each of the bubble in the $0 - x_0$ region (See Fig. 3).

Thus

$$P_a = \int_0^{x_0} \frac{N}{V} p(x) \, dx = \int_0^{T_0} N p(t) \, dt$$

(2.1)

The sound pressure at $Q$ can be expressed as

$$P(t) = P_a \exp[i \omega t]$$

(2.2)

where $\omega$—the center angular frequency of sound pressure at $Q$.

By doing Fourier transform to the $P(t)$, we can find the amplitude frequency spectrum of bubble group noise at $Q$ as (See Fig. 4)

$$A(\omega) = \left| \frac{P_a}{(\omega - \omega_0)} \right|$$

(2.3)

![Fig. 3 The sound pressure amplitude of bubble group at collapse](image)  ![Fig. 4 The amplitude frequency spectrum of bubble group noise](image)