THE ANALOGUE SIMULATION CRITERION OF THE CAVITATION

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Abstract

Employing the available theory, the analogue simulation criterion of the cavitation is introduced to provide the basis for the analogue simulation of the cavitation.

Key words cavitation, analogue, simulation criterion

I. The Origin of Question

By reasons of numerous factors and complicated relationships, the pure theoretical description and numerical calculation are very difficult in the cavitation research. To date, the main method is the test in the research of cavitation. Thus the analogue theory is very important.

In general cavitation model tests, the analogue simulation criterion of cavitation is the cavitation number

\[ K = \frac{\rho_m - \rho_v}{(\rho V_m^2)/2} \]  

(1.1)

where \( \rho_m \) is the pressure at infinity in the flow; \( V_m \) is the velocity at infinity in the flow; \( \rho \) is the density of liquid; \( \rho_v \) is the saturated vapor pressure of the liquid.

From equation (1.1), we can see, there are different combinations of \( V_m \) and \( \rho_m \) for the same value of \( K \). But the \( V_m \) is not equivalent to \( \rho_m \) as far as the cavitation. In practice, cavitation number is only a consistent form of similarity criterion of Euler number. Only cavitation number is not enough to finish the complete analogue of cavitation. The experimental investigation of cavitation noise was completed with a slender body of revolution in the water tunnel at Northwestern Polytechnical University[11]. Fig. 1 shows two noise curves measured with the same model, the different speeds and pressures of water, and the same cavitation number \( K \).

It is seen from Fig. 1 that there are great differences in the cavitation noise for the same cavitation number. The great differences of cavitation noise show that there are great differences in the cavitations. Namely, there exists no sole definite relationship between the cavitation
phenomenon and cavitation number. Thus, to find the new characteristic quantity or similarity criterion is one of the key problems in further investigations of the cavitation. In this paper, the analogue simulation criterion of the cavitation is introduced on the basis of the present theory.

II. The Motion Equation of a Single Bubble in a Free Flow Field

The basic assumption:

1) The bubble is considered as spherical in the process of movement.
2) The effect of the bubble on flow field may be neglected.
3) The changing process of gas inside the bubble can be regarded as the isothermic process.
4) Thermodynamic process is not considered.

Employing the basic assumption, the motion equation of a single bubble in the free flow field can be expressed as

\[ \frac{d\mathbf{v}}{dt} = \frac{3}{4R} (\mathbf{u} - \mathbf{v}) |\mathbf{u} - \mathbf{v}| \mathbf{C}_d + \frac{3(\mathbf{u} - \mathbf{v})}{R} \frac{dR}{dt} - \frac{3}{\rho} \nabla p + 2g \mathbf{k} \] (2.1)

(the center of mass motion equation of bubble)

\[ R \ddot{R} + \frac{3}{2} \dot{R}^2 - \frac{|\mathbf{u} - \mathbf{v}|^2}{4} = \frac{1}{\rho} \left( p_m - \frac{2\sigma}{R} - p_a \right) - \frac{4v \dot{R}}{R} \] (2.2)

(the radial motion equation of bubble)

\[ R^3 p_s - R^3 p_{s0} = \frac{6\sqrt{\pi/3} T_s B}{\mu_s} \int_{t_0}^{t} (C_s - C_1) \left( \frac{3D |\mathbf{u} - \mathbf{v}|}{2R} \right)^{1/2} R^2 dt \] (2.3)

(the gas diffusion equation)

\[ \rho \left( R \ddot{R} + \frac{3}{2} \dot{R}^2 \right) + J_{sv} \left( \frac{1}{\rho_s} - \frac{1}{\rho} \right) - \frac{d}{dt} \left( RJ_{sv} \right) + p_m - 4v \dot{R} \frac{\dot{R}}{R} - \frac{J_{sv}}{\rho} = 0 \] (2.4)

(the vapor-liquid phase change equation)

The initial condition and boundary condition

where \( \mathbf{v} \) — the center of mass velocity of bubble, \( R \) — the radius of bubble, \( \mathbf{u} \) — the velocity of liquid, \( C_d \) — the drag coefficient of bubble, \( p \) — the pressure of liquid, \( g \) — the acceleration of gravity, \( k \) — the plumb down unit vector, \( p_m \) — the mixed gas (vapor) pressure inside the bubble, \( \mathbf{P}_s, \mathbf{P}_{s0} \) — the gas partial pressure and its initial value inside the bubble, \( \sigma \) — the surface tension coefficient of liquid, \( \nu \) — the motion viscosity coefficient of liquid, \( T_s \) — the absolute temperature of gas inside the bubble, \( B \) — the gas universal constant, \( \mu_s \) — the molecular weight of gas, \( C_s \) — the average concentration of solution gas in liquid, \( C_1 \) — the concentration of solution gas in liquid at bubble-wall, \( D \) — the diffusion coefficient of gas, \( J_{sv} \) — the evaporation capacity of liquid, \( \rho_s \) — the density of gas.

III. The Analogue Simulation Criterion of the Motion State of a Bubble

The necessary and sufficient condition of similarity between the two phenomena is that the similarity of initial condition and boundary condition can satisfy identical differential equation.

Let the similar constant as follows (subscripts \( H \) represent parameters of prototype and \( m \) represent corresponding parameters of the model):