FINITE ELEMENT ANALYSIS FOR THE UNSTEADY NEARSHORE CIRCULATION DUE TO WAVE-CURRENT INTERACTION (I) — NUMERICAL MODEL

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Abstract

In this paper, a numerical model for predicting the unsteady nearshore circulation due to wave-current interaction was proposed. In addition to the traditional continuity, momentum and energy equations, the dispersion and refraction relations were included in the governing equations. Moreover, the effects of lateral shears, wind, radiation and bottom stresses were analysed in the governing equations. Therefore, we expect that this model may more completely and exactly reflect the law of wave-current interaction.

In part (II) we will adopt the selective lumping two-step explicit finite element method to solve the model, and some examples will be presented.

Key words numerical model, finite element method, unsteady circulation, nearshore circulation

I. Introduction

In recent years, there has evolved a great need for predicting wave-current interaction and the law of sediment transport in sea bed along with the development of marine resources. One of the important methods to solve the problem is to establish relative mathematical models, and to carry out their practical computer simulations, which have an important significance in the aspect of the ocean transport, the dock-building, the development of marine resources and the prevention and cure of ocean pollution. Therefore the countries with long-coastlines, such as Japan, attach importance to the work.

Up to now many authors proposed various numerical methods. By the finite difference method, Birkemeier and Dalrymple\(^\text{[1]}\) presented a model to predict the nearshore circulation due to wind and waves and Nada et al., proposed a model for the steady nearshore circulation. Recently Kawahara et al. suggested a finite element scheme which can simultaneously analyse waves, circulation and sediment transport. In this paper, another finite element model, which couples the refraction and dispersion relations in addition to the fundamental equations included Kawahara's, was proposed. Meanwhile, the stresses caused by ocean wind on the surface of waves, the bottom fraction stresses, the lateral shears stresses

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due to turbulence fluctuation, as well as the stresses of wave radiation were also included in the momentum equation.

The above work tried to obtain a more thorough and exactly numerical model which has been carried out by finite element method and coded with Fortran. Some calculated results are in good agreement with observations.

II. Governing Equations

1. Continuity equation

As shown in Fig. 1, \( \eta, h \) and \( H \) denote the water level, the water depth and the water height respectively. After averaging time over a wave period and integrating the water depth from \( z = -h(x, y, t) \) to \( z = \eta (x, y, t) \), we can obtain two-dimensional continuity equation by the law of mass conservation as follows

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(uD) + \frac{\partial}{\partial y}(vD) = 0
\]  

(2.1)

where \( D = \eta + h \) and \( u \) and \( v \) are \( x \) and \( y \) components of current velocity respectively.

![Fig. 1 Definition of \( \eta, h \) and \( H \)](image)

![Fig. 2 Definition of \( \theta \) and \( k \)](image)

2. Momentum equation

Integrating the depth and averaging time over a wave period then horizontal momentum equations of \( x \) and \( y \) components are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial \eta}{\partial x} = -\frac{1}{\rho} \frac{\partial \tau_x}{\partial y} - \frac{1}{\rho D} \left( \frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} \right)
\]

\[
+ \frac{1}{\rho D} \tau_{we} - \frac{1}{\rho D} \tau_{be}
\]  

(2.2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial \eta}{\partial y} = -\frac{1}{\rho} \frac{\partial \tau_y}{\partial x} - \frac{1}{\rho D} \left( \frac{\partial s_{yy}}{\partial x} + \frac{\partial s_{yx}}{\partial y} \right)
\]

\[
+ \frac{1}{\rho D} \tau_{we} - \frac{1}{\rho D} \tau_{be}
\]  

(2.3)

where \( \rho \) is the water density and \( g \) is the acceleration of gravity and \( \tau_x, \tau_y, \tau_{we} \) and \( \tau_{be} \) are concerned with the stresses of lateral shear, radiation, wind and bottom friction respectively.

2.1. Radiation stress

Longuet-Higgins\(^{[4]} \) has pointed out that for a progressive, linear, small amplitude wave, the radiation stress terms \( s_{xx}, s_{xy}, s_{yy} \) can be approximated by