On the Envelope Soliton near Critical Density in a Plasma with Cold Ions and Two-Temperature Electrons.

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Summary. — We have analysed the formation of envelope soliton near critical density in a plasma consisting of two-temperature electrons and cold ions. The non-linear Schrödinger-like equation obtained is \[ i \psi_t + p \psi_{xx} + q_1 |\psi|^4 \psi = 0 \] which we call the modified non-linear Schrödinger equation. It is also observed that this approach leads to a physical situation where a linear combination of both the modified and usual NLS equations holds, in the form \[ i \psi_t + p \psi_{xx} + q_1 |\psi|^2 \psi + q_2 |\psi|^4 \psi = 0. \] It is demonstrated through graphical analysis that \( q_1, q_2 \) thought of as a function of \( \beta (= T_e/T_{eh}) \), behave in opposite way. That is, when \( q_1 \) grows, \( q_2 \) decays, or vice versa. Lastly we demonstrate that this equation can sustain a type of solution other than the usual solitary profile. The form of such a wave is also depicted graphically.

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1. — Introduction.

Weakly non-linear waves in plasma in regimes where the linear dispersion relation gives a phase velocity nearly independent of the wave number can be described by the KdV equation, whose mathematical and physical properties have been thoroughly investigated. Of late various derivations of both the KdV and non-linear Schrödinger equations (NLS) have been advocated. Each such formulation has its own advantages and disadvantages. One important approach is that of reductive perturbation due to Washimi and Taniuti[1] and the other one is due to Fried and Ichikawa and Roy Chowdhury et al. [2]. While it is quite a laborious job to reduce the NLS equation in the former approach, the same can be done with much convenience in the latter one. On the other hand, an important problem that has been studied in the case of KdV situation is that of critical density—a region of transition from KdV to MKdV equation [3]. It is in this situation that the shock wave is generated, due to the vanishing of non-linearity in lower order of perturbation.

Unfortunately the same problem has drawn little attention in the NLS case. The
reason may be the enormous amount of computation involved in the reductive perturbation approach. In this paper we have formulated the problem of critical density in relation to the envelope soliton (the NLS situation) by formulating an extension of the Fried-Ichikawa approach. We have observed that a modified non-linear Schrödinger equation is generated (MNLS) with fifth power of non-linearity and we have analysed in detail the various kinds of waves that may be sustained by MNLS.

2. - Formulation of MNLS near critical density.

To start with we assume the existence of a suitable non-linear dispersion relation

\[ \varepsilon(k, W, A) = 0, \]

A being the amplitude. If initially the wave amplitude (in our case for the electrostatic potential \( \phi \)) is

\[ \phi(x, 0) = \int dk \phi_k \exp[ikx] + \text{c.c.} \]

with \( \phi_k \) peaked around \( k = k_0 \), then in the linear theory the long-time behaviour is given by

\[ \phi(x, t) = \int dk \phi_k \exp[i(kx - wt)] + \text{c.c.}. \]

Our derivation of the NLS equation rests on the assumption that we can continue to use (3) even in case of non-linear dispersion relation. We consider the case where both the amplitude and the spread of \( k \) values are small. So that an expansion of the dispersion relation, first in \( A^2 \), then in \( k' = k - k_0 \) is valid. So,

\[ \varepsilon(k, W, A) = \varepsilon(k, W, A_0) + A^2 \frac{\partial \varepsilon}{\partial A^2} + \ldots = 0. \]

For \( W \), we have

\[ W = \Omega(k) + MA^2 = W_0 + \Gamma \]

and

\[ \Gamma = v_g \sqrt{k} + v_g' \frac{\sqrt{k^2}}{2} + M A^2, \]

\( \Omega(k) \) being the solution of the linear dispersion relation,

\[ \begin{cases} \varepsilon[k, \Omega(k), 0] = 0, \\ W_0 = \Omega(k_0), \\ v_g = \Omega'(k_0), \\ v_g' = \Omega''(k_0), \\ M = \frac{\partial W}{\partial A^2} \mid_{k = k_0, W = W_0, A = 0}. \end{cases} \]