Transfer-function modelling of arteries

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Abstract—The paper describes a method for processing input/output blood velocity waveforms for femoral arteries obtained from ultrasonic Doppler flowmeters. Normal, diseased and early cases of disease are discussed, and it is shown that a pictorial display of the roots of a quadratic equation provide a simple quantitative assessment of progress and treatment of disease

Keywords—Artery, Data processing, Transfer function models, Ultrasonic Doppler flowmeters

1 Introduction

Blood-velocity/time waveforms recorded transcutaneously from the common femoral and popliteal arteries, using ultrasonic Doppler shift flow-velocity meters, have been used to classify the collateral circulation present in cases of complete occlusion of the superficial femoral artery (Woodcock, 1970, Fitzgerald et al., 1971, Morris et al., 1976). The waveforms consist of the maximum frequency envelope of the sonagram and are available

Fig. 1 Waveforms for normal case a

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Fig. 2 Waveforms for diseased case c
in digitised form. Visual inspection of the pairs of waveforms, called input and output for convenience, show distinctive differences between those obtained from normal arteries and those where the superficial femoral artery is completely blocked, and limb perfusion maintained by collateral circulation. Typical waveforms for cases (a) and (c) are shown in Figs. 1 and 2, a simple time delay and change of d.c. level in the outputs are omitted. In all normal cases the outputs show very marked oscillations, whereas in the diseased cases the oscillations are either very small in amplitude or nonexistent.

Typical waveforms for cases (a) and (c) are shown in Figs. 1 and 2, a simple time delay and change of d.c. level in the outputs are omitted. In all normal cases the outputs show very marked oscillations, whereas in the diseased cases the oscillations are either very small in amplitude or nonexistent.

However, early stages of disease are not easily detected by visual inspection of the waveforms alone. This is shown in Fig. 3, case (b), where the waveforms are similar to those of Fig. 1. A signal should be processed in whatever way will emphasise or assess its features (Sayers, 1970). The process which is described in this paper is likely to be useful not only for early detection of disease, but also for monitoring the effects of treatment.

2 Method of data processing

The first part of the method uses a standard multiple regression computer programme (ICL, 1973).

If \( x_k \) and \( y_k \) denote input and output at the \( k \)th sampling instant then, for example, a second order model relating the input and output can be expressed in the form

\[
y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2} + e_k \tag{1}
\]

where \( e_k \) is a small random modelling error of zero mean, due to nonlinearities and noise.

If

\[
E = \sum_k e_k^2
\]

then the multiple regression program produces those values of the parameters \( b_0, b_1, b_2, a_1 \) and \( a_2 \) which minimise \( E \).

Other orders of models can be used and were investigated, for example, a 3rd-order model

\[
y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} + b_3 x_{k-3} - a_1 y_{k-1} - a_2 y_{k-2} - a_3 y_{k-3} + e_k \tag{2}
\]

but it was found by examining the minimum value of \( E \) (Unbehauen and Gohring, 1973, Boom and Enden, 1973) obtained in each case that the second order model of eqn. 2 was suitable, and this is the model which has been used in this paper.

If the error term \( e_k \) is omitted then eqn. 1 is recognised as a finite-difference equation. This may very easily be converted into an approximate differential equation, provided the sampling time interval \( \Delta T \) is

\[
\begin{align*}
\text{Fig. 3} & \quad \text{Waveforms for early disease case b} \\
\text{Fig. 4} & \quad \text{Pole positions for cases studied}
\end{align*}
\]