A SCREW DISLOCATION BY NONLINEAR CONTINUUM MECHANICS*

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Abstract

Based on the nonlinear geometry field theory of continuum mechanics, this paper analyses the stress field due to a screw dislocation in an infinite medium. The results reveal the high-order effect of the stress field. When this effect is small, the result can be reduced to one of the classical linear elasticity. The body couple field of the screw dislocation is also investigated in this paper. The analytical expression of the body couple due to a screw dislocation is obtained with small rotation deformation. As the application of theoretical results, the stress and the body couple at the interface of the crystals are calculated when the screw dislocation is near the interface.

Key words nonlinear geometric field, screw dislocation, body couple

I. Introduction

The defects in solids and their effects on mechanical properties of solids are an important subject in solid physics, solid mechanics, metallurgy and material science. Macroscopic mechanical properties of solids are related to their microstructures especially crystal defects. To explain the macroscopic mechanical properties, it is necessary to investigate deeply microscopic mechanical properties of solids. A comprehensive description on this problem was given by Mura in his notable book "Micomechanics of defects in solids". In fact, a lot of defects arise in many engineering materials such as metal, alloy, rock, ceramic and composite material. These defects determine the mechanical properties of the materials. The mechanical properties are different from those of the practical measurement by several orders when defects are not taken into consideration, which shows that the solid defects play a critical role in mechanical properties. It requires a further investigation of the microstructure of materials containing defects. One of those theories about solid defects that have been studied most and that can best describe all mechanical properties of materials is none other than the dislocation theory.

In early this century, Volterra etc. proposed the concept of the dislocation in a continuous medium. In 1934, on the basis of experiments and observations of former, Taylor and some others put forward the conception and the picture of dislocations in crystals independently and respectively. They first connected dislocations from elasticity theory with dislocations in

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In 1938, Burgers set forth the concept of screw dislocation. In the fifties, Kröner et al. established and developed continuum dislocation theory. The dislocation theory came into continuum mechanics and became a part of it. Recently Duan et al. dealt with the dynamics of defects by a four-dimensional differential manifold theory.

A new theory of nonlinear continuum mechanics, which is based on the method of co-moving coordinates and the strain-rotation decomposition theorem, was first proposed by Prof. Chen in 1969. A recently published book "Rational Mechanics" states and sums up the theory and its applications. The new theory has been applied successfully in many fields such as in computational mechanics, fracture mechanics, experimental mechanics and biology mechanics. The new theory has been used to solve many problems and to explain many phenomena which can't be done by small deformation theory.

This paper investigates the rotation field of a screw dislocation in an infinite medium based on the nonlinear theory. First, a geometry model of a screw dislocation is established from its geometric character. Then, from the obtained model, the nonlinear geometry field of the screw dislocation is calculated in detail. The constitutive relation between the displacement and the stress is still in a form of Hooke's law because of the complicated nonlinear problem. The results show the high-effects of the nonlinear geometry field on the stress. The nonlinearity of the dislocation results from its fabrication and properties of the material. When deformation is small, the present result is identical with the classical one. With the constitutive relation of the rotation, the body couple due to a screw dislocation is investigated. Finally, the body couples at the interface are calculated when a dislocation is near the interface. The results provide a theoretical basis on the analysis of nonlinear displacement field due to defects. They also explain experimental phenomena of the cracks along the interface due to rotation of crystals.

II. Basic Theory

The nonlinear finite deformation theory based on Strain-Rotation decomposition theorem was first established and quickly developed by Prof. Chen Zhi-da, which have successfully solved the difficult mathematical and mechanical problem about how to define precisely a finite deformation state and local rotation of the neighboring area at a deformed body point. Therefore, it has been widely used in such fields as experimental mechanics, computational mechanics, fracture mechanics and biology mechanics and so on.

The new theory has two characteristics. One is that the Euler movable coordinate system is extended to the co-moving coordinate system which describes the motion of the deformed bodies. In finite deformation condition, the co-moving coordinate system can definitely reveal relations of the body before and after deformation. Muranghan, Brillouin, Synge and Chien Wei-zang et al. have introduced the co-moving coordinates in analyses of finite deformation. The numerical calculations and experiments have proved that the application of the co-moving coordinate system is superior to that of any other coordinate system. The other characteristic of the new theory is employment of summation of the symmetric strain tensor and the orthogonal rotation tensor instead of traditional polar decomposition to describe finite deformation of a body. The Strain-Rotation decomposition overcomes the un-uniformity of the polar decomposition. The strain tensor thus becomes more precise and direct. The calculation by nonlinear finite element has shown the advantages of S-R decomposition.

When the co-moving coordinates are used, first choose a fixed coordinate frame \( \{X^i\} \) in