NONLINEAR THREE-DIMENSION ANALYSIS FOR AXIALLY SYMMETRICAL CIRCULAR PLATES AND MULTILAYERED PLATES

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Abstract

Analytic nonlinear three-dimension solutions are presented for axially symmetrical homogeneous isotropic circular plates and multilayered plates with rigidly clamped boundary conditions and under transverse load. The geometric nonlinearity from a moderately large deflection is considered. A developmental perturbation method is used to solve the complicated nonlinear three-dimension differential equations of equilibrium. The basic idea of this perturbation method is using the two-dimension solutions as a basic form of the corresponding three-dimension solutions, and then processing the perturbation procedure to obtain the three-dimension perturbation solutions. The nonlinear three-dimension results in analytic expressions and in numerical forms for ordinary plates and multilayered plates are presented. All of the plate stresses are shown in figures. The results show that this perturbation method used to analyse nonlinear three-dimension problems of plates is effective.

Key words three-dimension analysis, geometric nonlinearity, perturbation method, axially symmetry

I. Introduction

The high development of science and technology requires the use of the three-dimension theory of elasticity for obtaining highly accurate predictions of characteristics of plates, especially sandwich plates and laminated plates, etc. To this kind of plates, the delamination caused by transverse normal stress and transverse shear stress is a main model of failure. Although the two-dimension analysis of plates may be easily found in books of mechanics of elasticity, there are a few of three-dimension analyses, especially nonlinear three-dimension analyses, Timoshenko presented the linear three-dimension solution of axially symmetrical homogeneous isotropic circular plates only with stress boundary conditions, but the solution cannot satisfy the displacement boundary conditions. Vlasov offered the linear three-dimension solution of simply supported isotropic plates. However, there is almost no nonlinear three-dimension analysis of ordinary plates and laminated plates. This paper gives the nonlinear three-dimension solutions of clamped homogeneous isotropic plates and laminated plates subjected to uniform transverse loading. The perturbation method used to obtain the solutions is easy and effective, it is a suitable method for the nonlinear three-dimension analysis of plates.
II. The Nonlinear Three-Dimension Analysis of Ordinary Plates

Many problems in stress analysis which are of practical importance are concerned with a solid of revolution deformed symmetrically with respect to the axis of revolution. For problems of this kind it is often convenient to use cylindrical coordinates (see Fig. 1). The deformation being symmetrical with respect to the $z$-axis, it follows that the components of stresses, strains and displacements are independent of the angle $\theta$, and all derivatives with respect to $\theta$ vanish. The components of shear stress $\tau_{y\theta}$, $\tau_{z\theta}$ and the tangential component of displacements $v$ also vanish on account of the symmetry. If the body forces are neglected, $u$ and $w$ are the components of displacements in the radial- and $z$-directions, respectively. And the plate is made of homogeneous isotropic material with the modulus of elasticity $E$ and the Poisson's ratio $\nu$.

![Fig. 1 The coordinate system and shape of circular plates.](image)

Considering this kind of nonlinearity: the linear strains $\varepsilon_{ij}$ and the squares of angles of rotation $\omega_1^2$ of an element of the plates are small compared with unity, and the Hooke's law is suitable. That is, the plate is of moderately large deflections. And in most engineering applications of plates, the thickness to span ratio of plates is very small and the plate is a massive body in its plane, the rotation about an axis normal to the plane is generally much smaller than those about axes in the plane of the plate. Therefore, the rotation $\omega_z$ equals zero. In the case of the circular plate being of axially symmetrical deformation, the following strain-displacement relations can be obtained

$$
\begin{align*}
\varepsilon_r &= u_r + \frac{w_z^2}{2}, \\
\varepsilon_\theta &= u_\theta, \\
\varepsilon_z &= w_z + \frac{u_z^2}{2},
\end{align*}
$$

in which a comma denotes partial differentiation with respect to the corresponding coordinate.

And the Hooke's law is

$$
\begin{align*}
\sigma_r &= E_1(\mu \varepsilon_r + \varepsilon_r) \\
\sigma_\theta &= E_1(\mu \varepsilon_\theta + \varepsilon_\theta) \\
\sigma_z &= E_1(\mu \varepsilon_z + \varepsilon_z) \\
\tau_{rz} &= E_1 \gamma_{rz}/2
\end{align*}
$$

where

$$
E_1 = E/(1+\nu), \quad \mu = \nu/(1-2\nu)
$$

The nonlinear three-dimension differential equations of equilibrium are

$$
\begin{align*}
(\sigma_r + u_r \tau_{rz})_r + (\tau_{rz} + w_z \sigma_z) + (\sigma_r - \sigma_\theta + u_z \tau_{rz})/r &= 0, \\
(\tau_{rz} + w_z \sigma_r)_r + (\sigma_z + w_z \sigma_z) + (\tau_{rz} + w_z \sigma_r)/r &= 0
\end{align*}
$$

Based on the perturbation method, the nondimension parameters are introduced as follows