Representation of the Anelastic Behaviour of a Solid Having a Gaussian Relaxation Spectrum.

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Summary. — The approximate representation obtained in a previous paper for the frequency dependence of the energy dissipation taking place in a vibrating solid with a Gaussian relaxation spectrum has been extended, for the same spectrum, to the frequency dependence of elastic modulus as well as to the time dependence of after-effect and creep. The procedure followed in order to obtain these representations appears susceptible of application to other types of spectra.

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1. — Introduction.

In a previous paper (1), the coefficient \( Q^{-1} \) which characterizes the dissipation of elastic energy taking place in a vibrating anelastic solid, when a Gaussian spectrum of relaxation effects is present, was expressed in closed form by means of elementary functions with an accuracy sufficient for the interpretation of experimental results. It has been suggested (2) that it would be useful to derive similar expressions also for the frequency dependence of the elastic modulus and for the time dependence of the strain produced by the sudden application of a stress (creep) or by the removal of the same stress (after-effect).

(2) The authors express their gratitude to Prof. A. Seeger for this suggestion.
In the present paper, a procedure has been devised in order to derive expressions which contain, in closed form, only elementary functions and represent the frequency dependence of modulus and dissipation, as well as the time dependence of creep and after-effect. As in the paper already quoted, the representation has been extended only to the values of physical interest and its accuracy has been limited to that required by the interpretation of experimental data.

This procedure has been applied, for the moment, only to Gaussian spectra, but its general character seems to indicate the possibility of further applications to other types of spectra, at least to those spectra whose width and shape depend upon the values of a parameter in a way similar to that of Gaussian spectra.

2. Notation.

For a Gaussian time spectrum, the density \( \sigma_0 \) of the relaxation strength \( S \), normalized with respect to the total value of \( S \), is given by

\[
\sigma_0 = \frac{1}{\beta \sqrt{\pi}} \exp \left[ -\frac{x^2}{\beta^2} \right],
\]

where \( x = \ln \frac{\tau}{\tau_m} \), \( \tau \) is the generic relaxation time, \( \tau_m \) is the time corresponding to the centre of the spectrum, \( \beta \) is a parameter which characterizes the distribution of strength with respect to time.

When a solid with such a spectrum vibrates on one of its fundamental modes, with angular frequency \( \omega \), one finds that the elastic modulus \( M(\omega) \), involved in the vibration mode considered, varies with frequency from its highest value \( M_v \) (unrelaxed) corresponding to \( \omega \to \infty \) to its lowest value \( M_R \) (relaxed), corresponding to \( \omega \to 0 \). By dividing the difference \( M(\omega) - M_R \) by the total modulus excursion \( M_v - M_R \), one obtains the (partially) normalized function \( f_1(0, 1) \) which characterizes the frequency dependence of modulus.

In a similar way, when \( Q^{-1} \) is measured as a function of \( \omega \) and is divided by its maximum value \( Q_v^{-1} \), one obtains the (partially) normalized function \( f_2(0, 1) \) which characterizes the frequency dependence of dissipation.

Finally, when the coefficient \( \varepsilon \) of some simple type of strain (extensional, torsional and so on) is measured as a function of time \( t \), after the sudden application at \( t = 0 \) of a corresponding simple stress, constant for \( t > 0 \), it is found that \( \varepsilon(t) \) varies from its lowest (instantaneous or unrelaxed) value \( \varepsilon_v \), corresponding to \( t = 0^+ \), to its highest (final or relaxed) value \( \varepsilon_R \), corresponding

\(^{(\#)}\): Completely normalized functions, i.e. normalized with respect to both the dependent and independent variables, will be introduced in the following.