A THEORETICAL AND EXPERIMENTAL INVESTIGATION
OF A PRIMARY RESONANCE OF A THREE CIRCULAR
PLATES TORSION VIBRATION SYSTEM*

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Abstract

The method of averaging is applied in this paper to deal with a primary resonance
of a three circular plates torsion vibration system having cubic nonlinearities which is
excited by a simple-harmonic excitation. Bifurcation equation of the steady state
response is obtained and its singularity analysis is given. The results of theoretical
analysis are shown to be in good agreement with experimental ones.

Key words three circular plates torsion vibration, method of averaging,
primary resonance, singularity theory, jump

I. Introduction

Rotor-shafting is widely used in engineering, and rotor-shafting torsion vibration is
gaining attention widely. In this paper, the model of the rotor-shafting torsion vibration
of a 200MW turbogeneration set is designed and a three circular plates torsion vibration
system with cubic nonlinearities to a simple-harmonic excitation is studied. The objective of
the paper is to study the dynamics phenomena of the system while \( \Omega = \omega_2 \). The method
of averaging is used and the steady state response bifurcation equation and its singularity analysis
are given. The transition variety and bifurcation diagrams of the bifurcation equation are
obtained. The paper points out the existence of jump phenomenon in response curves. The
experimental results confirm that the force-response curves have two different topological
structures on two different physical parameter regions. The conclusion is shown to be in good
agreement with singularity theoretical analysis.

II. Mathematical Model

Figure 1 shows the mechanical model of the three circular plates torsion system. Three
circular plates are joined by the springs constituting a rotor-shafting. In the Figure 1, \( I_i \)
represents the rotation inertia of the \( i \)th circular, \( \phi_i \) represents the torsion angular of the \( i \)th
circular, \( \mu_{12}, \mu_{23} \) are damping coefficients, \( K_{12}, K_{23} \) are linear torsion stiffness, \( K \) is cubic
nonlinearity stiffness, and \( F_{\cos \omega_2 t} \) is a harmonic external excitation. Kinetic energy \( T \),

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potential energy $V$ and dissipation function $R$ can be obtained from Figure 1.

\[ V = \frac{1}{2} K_{12} (\phi_1 - \phi_2)^2 + \frac{1}{2} K_{23} (\phi_2 - \phi_3)^2 + \varepsilon (\phi_1 - \phi_2)^4 \]

\[ R = \varepsilon (\phi_1 - \phi_2)^2 + \frac{1}{2} \mu_{12} (\phi_1 - \phi_2) + \frac{1}{2} \mu_{23} (\phi_2 - \phi_3) \]

where $\varepsilon$ is a dimensionless parameter. After introducing Lagrange function $L = T - V$ and substituting it into the Lagrange equation, we obtain

\[ I_1 \ddot{\phi}_1 + I_{12} (\phi_1 - \phi_2) + \varepsilon \mu_{12} (\phi_1 - \phi_2) + \varepsilon K (\phi_1 - \phi_2)^3 = 0 \]

\[ I_2 \ddot{\phi}_2 - K_{12} (\phi_1 - \phi_2) + K_{23} (\phi_2 - \phi_3) - \varepsilon \mu_{12} (\phi_1 - \phi_2) + \varepsilon \mu_{23} (\phi_2 - \phi_3) \]

\[ - \varepsilon K (\phi_1 - \phi_2)^3 = \varepsilon F \cos \omega t \]

\[ I_3 \ddot{\phi}_3 - K_{23} (\phi_2 - \phi_3) - \varepsilon \mu_{23} (\phi_2 - \phi_3) = 0 \]

The torsion vibration equation of the three circular plates torsion system is obtained. Let $a_1 = \dot{\phi}_1, a_2 = \phi_2 - \phi_3$, the equation (2.1) can be transformed into the equation (2.2) about relative torsion vibration angular $a_1$

\[ a_1 + \frac{I_1 + I_2}{I_1 I_2} K_{12} a_1 - \frac{K_{23}}{I_2} a_2 = \frac{\varepsilon}{I_1 I_2} \left[ - (I_1 + I_2) \mu_{12} a_1 - (I_1 + I_2) K a_1 \right] + I_1 \mu_{23} a_2 - I_1 F \cos \omega t \]

\[ a_2 - \frac{K_{12}}{I_2} a_1 + \frac{I_2 + I_3}{I_2 I_3} K_{23} a_2 = \frac{\varepsilon}{I_2 I_3} \left[ I_3 \mu_{12} a_1 - (I_2 + I_3) \mu_{23} a_2 \right] + I_3 \mu_{23} a_1 + I_3 F \cos \omega t \]

Equations (2.2) can be used to analyze vibration problems of the three circular plates torsion vibration system.

The equations of undamped free vibration equations of the system are

\[ a_1 + \frac{I_1 + I_2}{I_1 I_2} K_{12} a_1 - \frac{K_{23}}{I_2} a_2 = 0, \quad a_2 - \frac{K_{12}}{I_2} a_1 + \frac{I_2 + I_3}{I_2 I_3} K_{23} a_2 = 0 \]

and corresponding to frequency equation is

\[ \omega^4 - \omega^2 (a_1 K_{12} + a_4 K_{23}) + a_1 a_4 K_{12} K_{23} - a_2 a_2 K_{12} K_{23} = 0 \]

Hence, we obtain

\[ (a_1 a_4 - a_2 a_2) K_{12} K_{23} = \omega^2, \quad a_1 K_{12} + a_4 K_{23} = \omega^2 + \omega^4 \]