THE FREE-INTERFACE METHOD OF COMPONENT MODE SYNTHESIS
FOR SYSTEMS WITH VISCOUS DAMPING*

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(Received Sep. 10, 1990; Communicated by Zhang Ru-qin)

Abstract

This paper presents a new free-interface method of component mode synthesis for linear systems with arbitrary viscous damping. The left and right projection matrices described by state-variable vectors are first introduced for components with rigid-body freedom. The operator function of projection matrices for state displacement and state force is proved, and then the state residual flexibility matrix and the state residual inertia-relief attachment mode are defined and employed. The results of three examples demonstrate that the method proposed in this paper leads to very accurate system eigenvalues and high mode-synthesis efficiency.

Key words  viscously damped systems, component mode synthesis, projection matrix

I. Introduction

It is more reasonable to assume viscous damping than to assume proportional damping for most engineering structures. The traditional component mode synthesis (CMS) is not suited to complex structures with viscous damping. In recent years, several papers on the application of CMS methods to structures with viscous damping have appeared [1-4]. Ref. [1] and [2] adopted complex component modes and extended the Craig-Bampton method to damped systems with symmetric substructure matrices by different patterns. Ref. [3] employed truncated complex component modes and a set of generalized real attachment modes fitting free-interface components. However, the calculation results are not accurate enough. Ref. [4] presented a new free-interface method of component mode synthesis which extended the Craig-Chang method to damped systems. However, it considered only the case that the eigen-subspace corresponding to zero eigenvalue of components is not defective, and gave only one example in which the damping matrices are symmetric.

In this paper the left and right projection matrices are introduced for components with rigid-body freedom. The operator function of projection matrices for state displacement and state force is reproved for the nondefective case and the defective case, respectively. The state residual flexibility matrix and the state residual inertia-relief attachment mode are defined and employed, which extends the Craig-Chang method fitting undamped systems successfully to arbitrary viscously damped systems. The examples given in the end of the paper cover the cases that the eigen-subspace

* Project supported by the Science Foundation of Xi'an Jiaotong University
corresponding to zero eigenvalue of components is either nondefective or defective, and cover both symmetric damping case and asymmetric damping case. These examples demonstrate that the method proposed in this paper leads to very accurate calculation results and high mode-synthesis efficiency due to the deleted complex modes information acquired by the defined state residual modes.

II. The Projection Matrices in the State Space

The state-space form of the equation of free vibration for a typical free-interface component may be written

\[ A \dot{X} + BX = F \]  \hspace{1cm} (2.1)

where

\[ A = \begin{bmatrix} O & m \\ m \ & c \end{bmatrix}, \quad B = \begin{bmatrix} -m & O \\ O & k \end{bmatrix}, \quad X = \begin{bmatrix} \dot{x} \\ x \end{bmatrix}, \quad F = \begin{bmatrix} O \\ f \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ f_i \end{bmatrix} \]  \hspace{1cm} (2.2)

where \( m, c \) and \( k \) are the mass, damping, and stiffness matrices, respectively. If the component has rigid-body freedom, \( k \) will be singular. The subscripts \( i \) and \( b \) denote the interior coordinates and the interface, or boundary, coordinates of displacement vector \( x \). \( f_i \) is the interface force from adjacent components. \( X \) and \( F \) are referred to as the state displacement and the state force, respectively. If not all matrices \( m, c, \) and \( k \) are symmetric, there is an adjoint state equation

\[-A^T \dot{Y} + B^T Y = F^*\]  \hspace{1cm} (2.3)

where the adjoint state displacement \( Y \) and adjoint state force \( F^* \) are given by

\[ Y = [\dot{y}^p, \quad y^p]^T, \quad F^* = [O, \quad (f^*)^T]^T \]  \hspace{1cm} (2.4)

The eigenproblems in accordance with Eqs. (2.1) and (2.3) are as follows

\[ B\Phi = -A\Phi \Lambda, \quad \psi^T B = -\Lambda \psi^T A \]  \hspace{1cm} (2.5)

where \( \Phi \) and \( \psi \) are the right and left complex mode matrices, respectively, and \( \Lambda \) is the complex spectrum matrix. It follows from Eq. (2.5) that

\[ \bar{B} = -\bar{A} \Lambda = -\Lambda \bar{A} \]  \hspace{1cm} (2.6)

where

\[ \bar{B} = \psi^T B \Phi, \quad \bar{A} = \psi^T A \Phi \]  \hspace{1cm} (2.7)

The left and right complex modes associated with different eigenvalues have the biorthogonality relationships.

The complex mode matrices \( \Phi \) and \( \psi \) of component with rigid-body freedom may be partitioned as

\[ \Phi = [\Phi_r, \quad \Phi_f], \quad \psi = [\psi_r, \quad \psi_f] \]  \hspace{1cm} (2.8)

where subscripts \( r \) and \( f \) denote the rigid-body mode partition and the flexible complex mode partition, respectively. Then Eq. (2.7) can be written as

\[ \bar{B} = \text{diag}[\bar{B}_r, \quad \bar{B}_f], \quad \bar{A} = \text{diag}[\bar{A}_r, \quad \bar{A}_f] \]  \hspace{1cm} (2.9)

where