THE END AND SPACING EFFECT OF INDUCTION COIL OF FINITE LENGTH ON THE EM FIELD IN HF PLASMA

Tang Fu-lin (唐福林) Chen Yun-ming (陈允明)
(Institute of Mechanics, Academy of Sciences, Beijing)
(Received July 28, 1987; Communicated by Dai Shi-qiang)

Abstract

In practical applications the diameter and height of the induction coil have the same order of magnitude, hence the end effect is unnegligible in HF (high frequency) plasma theory. The present paper calculates the electromagnetic field under the assumption of infinite plasma column with uniform conductivity. The results show that the magnetic induction differs greatly from that in vacuum case at axis, and even the direction is reversed in some cases. In contrast, the difference is not large at plasma surface, and the phase lag between B and E there changes little, either. Hence, the EM field at the surface in vacuum case can be more adequately applied as boundary condition in HF plasma computation.

I. Introduction

The HF plasma has recently seen wide application in high grade chemical products[11], plasma etching and plasma-enhanced chemical vaporization deposition, and thus promotes numerous studies in this field[21]. In HF plasma study, electromagnetic variables are important parameters, affecting not only energy coupling, plasma flow field and temperature profile, but the matching with external circuit as well. In HF plasma theories and computations available up to now, either the end effect of the induction coil of finite length was neglected[10], or one-dimensional electromagnetic equations were adopted and the $H_r$ profile produced by coil in vacuum at axis was taken as boundary condition[14]. In practice the diameter and length of the induction coil have the same order of magnitude, thus the radial component of magnetic field and the shielding effect of plasma on magnetic induction profile are unnegligible. In addition, the space between two adjacent turns has the same order of magnitude as the height of one turn. Hence the end effect is an urgent problem in HF plasma study. In the present paper the magnetic field of induction coil of finite length was calculated under the assumption of infinite plasma column with uniform electric conductivity. The results showed that the magnetic induction profile changed greatly at axis when plasma was present, even reverse magnetic field occurred in some cases. The magnetic lines of force were shown as well as profiles of the radial and axial components and phase lag along axis and radius. This will shed light on the characteristics of HF plasma and the boundary condition in its numerical simulations.

II. Equations

Vector potential $A_i$ was introduced inside plasma whose relation to magnetic induction $B$, was as follows
\[ \nabla \times \mathbf{A}_i = \mathbf{B}_i, \quad (2.1) \]

with \( \nabla \cdot \mathbf{A}_i = 0 \) as its normalization condition. From Faraday's law the electric field intensity \( \mathbf{E}_i \), was related to \( \mathbf{B}_i \) as

\[ \nabla \times \mathbf{E}_i = -\frac{\partial \mathbf{B}_i}{\partial t}, \quad (2.2) \]

In HF plasma \( \mathbf{E}_i \) was produced by HF electric current through induction coil, therefore

\[ \nabla \cdot \mathbf{E}_i = 0, \quad (2.3) \]

From (2.2) and (2.3) it follows

\[ \mathbf{E}_i = -\frac{\partial \mathbf{A}_i}{\partial t}, \quad (2.4) \]

The displacement current was negligible with frequency of the order of \( 10^6 \), and the velocity was not large, hence the Ampere's and Ohm's laws read as

\[ \nabla \times \mathbf{B}_i = \mu \mathbf{J}_i, \quad \mathbf{J}_i = \sigma \mathbf{E}_i, \]

where \( \mathbf{J}_i \) was current in plasma. \( \sigma \) and \( \mu \) stood for the electric conductivity and magnetic permeability, respectively. From (2.1) and (2.4) it followed

\[ \nabla^2 \mathbf{A}_i = \mu \sigma \frac{\partial \mathbf{A}_i}{\partial t}, \quad (2.5) \]

Letting \( \omega \) be the frequency, eq. (2.5) became

\[ \nabla^2 \mathbf{A}_i = i \omega \mu \sigma \mathbf{A}_i, \quad (2.6) \]

In a symmetric configuration shown in Fig. 1, \( \mathbf{A}_i \) and \( \mathbf{E}_i \) had only \( \varphi \)-component, and \( \mathbf{B}_i \) had only radial and axial components in cylindrical coordinates, e.g.

\[ \begin{align*}
\mathbf{A}_i(\rho, z) &= (0, A_{i\varphi}, 0) \\
\mathbf{E}_i(\rho, z) &= (0, E_{i\varphi}, 0) \\
\mathbf{B}_i(\rho, z) &= (B_{i\rho}, 0, B_{i\varphi})
\end{align*} \quad (2.7) \]

Therefore the \( \varphi \)-component of equation (2.6) could be written as

\[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial A_{i\varphi}}{\partial \rho} \right) + \frac{\partial^2 A_{i\varphi}}{\partial z^2} - \frac{A_{i\varphi}}{\rho^2} = i \omega \mu \sigma A_{i\varphi}, \quad (2.8) \]

Outside the plasma the vector potential \( \mathbf{A}_e \) consisted of that produced by current inside plasma \( \mathbf{A}_{e1} \) and that produced by current through induction coil \( \mathbf{A}_{e2} \), e.g.

\[ \mathbf{A}_e = \mathbf{A}_{e1} + \mathbf{A}_{e2}, \quad (2.9) \]

Based on similar reasoning the equation for \( \mathbf{A}_{e1} \) was