THEORY OF THICK-WALLED SHELLS AND ITS APPLICATION
IN CYLINDRICAL SHELL

Fang Ying-guang (房背光)

(Guangdong Inst. of Tech., Guangzhou)
Pan Ji-hao (潘纪浩) Chen Wei-xin (陈维新)

(East China Univ. of Chemical Tech., Shanghai)

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Abstract

In this paper, a theory of thick-walled shells is established by means of Hellinger-Reissner's variational principle, with displacement and stress assumptions. The displacements are expanded into power series of the thickness coordinate. Only the first four and the first three terms are used for the displacements parallel and normal to the middle surface respectively. The normal extruding and transverse shear stresses are assumed to be cubic polynomials and to satisfy the boundary stress conditions on the outer and inner surfaces of the shell. The governing equations and boundary conditions are derived by means of variational principle. As an example, a thick-walled cylindrical shell is discussed with the theory proposed. Furthermore, a photoelastic experiment has been carried out, and the results are in fair agreement with the computations.

Key words thick-walled shell, cylindrical shell, normal extruding stress; variational principle, photoelastic experiment

1. Introduction

Thick-walled shells are often used in various engineering fields. A number of theories for shells of moderate thickness have been proposed by several scholars [1-4]. As an improvement, the influence of extruding and transverse shear are considered to a certain degree, in order to fit the analysis for thicker shells. For much thicker shells, these theories are still unsuitable. Of course, the thick-walled shells may be treated as three-dimensional elastic bodies [5], but in general, it is very difficult to obtain exact analytical solutions because of excessive complexity. In this paper, thick-walled shells are treated as a two-dimensional problem. The displacements and stress distributions are assumed to be polynomials along the thickness. Thus the analysis is much simplified, and solutions can be obtained more easily. The numerical example shows that the theoretical results are in good agreement with the experiment.

II. The Fundamental Theory of Thick-Walled Shells

(1) Assumptions of displacements and stresses
In Fig. 1 a shell element is shown, where $\alpha$, $\beta$, $\gamma$ are orthogonal curvilinear coordinates. $p^+_i$, $p^-_i$, $p^+_z$, $p^-_z$ are loadings applied on the top and bottom surfaces. Assume the displacements can be expanded in series of $\gamma$, and the first four and first three terms are retained for displacements $u$, $v$ and $w$ respectively.

\[
\begin{align*}
u &= u_0(\alpha, \beta) + \gamma u_1(\alpha, \beta) + \gamma^2 u_2(\alpha, \beta) + \gamma^3 u_3(\alpha, \beta) \\
v &= v_0(\alpha, \beta) + \gamma v_1(\alpha, \beta) + \gamma^2 v_2(\alpha, \beta) + \gamma^3 v_3(\alpha, \beta) \\
w &= w_0(\alpha, \beta) + \gamma w_1(\alpha, \beta) + \gamma^2 w_2(\alpha, \beta)
\end{align*}
\]

(2.1)

where coefficients of each power of $\gamma$ are relatively independent functions. Because the influence of normal extruding and transverse shear stresses are important for thick-walled shells, we should make them meet the stress conditions on the top and bottom surfaces. These conditions are (Fig. 1):

\[
\sigma_3(\alpha, \beta, \pm h/2) = \left\{ \begin{array}{l} p^+_3 \\ p^-_3 \end{array} \right. \\
\tau_{13}(\alpha, \beta, \pm h/2) = \left\{ \begin{array}{l} p^+_3 \\ p^-_3 \end{array} \right.
\]

(2.2)

Assume

\[
\begin{align*}
(1+k_1\gamma)(1+k_2\gamma)\sigma_3 &= \omega^*_3(\alpha, \beta) + \gamma \omega^*_3(\alpha, \beta) \\
&\quad + \gamma^2 \omega^*_3(\alpha, \beta) + \gamma^3 \omega^*_3(\alpha, \beta) \\
(1+k_1\gamma)\tau_{13} &= \varphi^*_3(\alpha, \beta) + \gamma \varphi^*_3(\alpha, \beta) + \gamma^2 \varphi^*_3(\alpha, \beta) + \gamma^3 \varphi^*_3(\alpha, \beta) \\
(1+k_1\gamma)\tau_{23} &= \psi^*_3(\alpha, \beta) + \gamma \psi^*_3(\alpha, \beta) + \gamma^2 \psi^*_3(\alpha, \beta) + \gamma^3 \psi^*_3(\alpha, \beta)
\end{align*}
\]

(2.3a)

After making stresses $\sigma_3$, $\tau_{13}$, $\tau_{23}$ satisfy condition (2.2), we get the stresses in the form

\[
\begin{align*}(1+k_1\gamma)(1+k_2\gamma)\sigma_3 &= \left[(\omega_1(\alpha, \beta) + \gamma \omega_2(\alpha, \beta)) \left(1 - \frac{4\gamma^2}{h^2}\right)\right] \\
&\quad + \frac{1}{2} (p^+_3 H^+ - p^-_3 H^-) + \frac{3}{h} (p^+_3 H^+ + p^-_3 H^-) \\
(1+k_1\gamma)\tau_{13} &= \left[\varphi_1(\alpha, \beta) + \gamma \varphi_2(\alpha, \beta) \left(1 - \frac{4\gamma^2}{h^2}\right)\right] \\
&\quad + \frac{1}{2} (p^+_3 H^+ - p^-_3 H^-) + \frac{3}{h} (p^+_3 H^+ + p^-_3 H^-) \\
(1+k_1\gamma)\tau_{23} &= \left[\psi_1(\alpha, \beta) + \gamma \psi_2(\alpha, \beta) \left(1 - \frac{4\gamma^2}{h^2}\right)\right] \\
&\quad + \frac{1}{2} (p^+_3 H^+ - p^-_3 H^-) + \frac{3}{h} (p^+_3 H^+ + p^-_3 H^-)
\end{align*}
\]

(2.3b)