Nonlinear Index of Refraction in Semiconductors.

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(ricevuto il 9 Febbraio 1984)

Summary. — Experimental work on light refraction in semiconductors suggests that there are two separate mechanisms by which the index of refraction becomes nonlinear: direct saturation, on the one hand, and the Burstein-Moss effect, on the other hand. We show that this picture is incorrect, since it was based on the usage of the Bloch equation. Electron systems are not Boltzmann systems and the appropriate Fermi-Dirac master equation leads to a saturation that differs from the saturation of the Bloch equation. We show, using a simple two-level system, that this result may look like the spin result in one extreme (phonon bottleneck saturation), while it shows the features of the Burstein-Moss effect in the limit of small degeneracy of the upper level (Fermi saturation).

PACS. 78.20. — Optical properties and materials.

1. — Introduction.

In a recent paper on the optical properties of semiconductors by Wherrett and Higgins (1), the authors point out that there are two separate mechanisms by which a semiconductor can exhibit a nonlinear index of refraction. They explain that direct saturation is present in gallium arsenide, while a different mechanism determines the behaviour in indium antimonide. We show in this paper that, by using the correct master equation, one encompasses both cases. The misunderstanding results from the fact that the saturation

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formulae were taken from spin systems and that the present situation deals with a Fermi-Dirac system rather than a Boltzmann system. An explicit calculation is given for the two-level system.

The reader is reminded that in the nonlinear case the Kramers-Kronig relations can no longer be utilized.

2. – The Damping-saturation model.

In order to describe a system that is perturbed by an oscillatory field, one introduces the equation of motion for the density matrix $\rho$:

$$\frac{d\rho}{dt} = i\frac{\hbar}{\hbar} [H, \rho] - \frac{1}{2} [I', I'; (\rho - \bar{\rho})].$$

See, for instance, ref. (2). Equation (1), when written out in explicit matrix notation, represents two equations. The equation for the off-diagonal matrix elements is given by

$$\frac{d\rho_{21}}{dt} = -i\omega_0 \rho_{21} + \frac{\mu E}{\hbar} (\rho_{11} - \rho_{22}) - \frac{\rho_{21}}{T_2},$$

where the first term stems from the unperturbed Hamiltonian, the next term is due to the «pump», i.e. the coupling Hamiltonian to the oscillatory field, and the last term is the damping due to coupling to the other systems. This term is called the transverse relaxation time and gives rise to the line broadening. Specifically, this relaxation would be due to spin-spin interaction in spin systems or to electron-electron interaction in electron systems.

To determine the behaviour of the diagonal elements of the density matrix, one needs to solve

$$\frac{d(\rho_{11} - \rho_{22})}{dt} = 2i \frac{\mu E}{\hbar} (\rho_{21} - \rho_{12}^*) - \frac{\Delta(\rho_{11} - \rho_{22})}{\tau}.$$