HETEROCLINIC ORBIT AND SUBHARMONIC BIFURCATIONS
AND CHAOS OF NONLINEAR OSCILLATOR

Zhang Wei (张 伟) Huo Quan-zhong (霍拳忠)

(Tianjin University, Tianjin)

Li Li (李 鹏)

(Peking Polytechnic University, Beijing).

(Received Jan. 21, 1991)

Abstract

Dynamical behavior of nonlinear oscillator under combined parametric and forcing excitation, which includes van der Pol damping, is very complex. In this paper, Melnikov's method is used to study the heteroclinic orbit bifurcations, subharmonic bifurcations and chaos in this system. Smale horseshoes and chaotic motions can occur from odd subharmonic bifurcation of infinite order in this system for various resonant cases. Finally the numerical computing method is used to study chaotic motions of this system. The results achieved reveal some new phenomena.

Key words  heteroclinic orbit bifurcations, subharmonic bifurcations, chaotic motions, parametric excitation, Melnikov's method

I. Introduction

The studies for bifurcations and chaos of nonlinear dynamical systems under combined parametric and forcing excitation are one of the most interesting areas in the theory of nonlinear oscillations. These systems can exhibit extremely complex behavior patterns and stability characters. The explanation of this complex behavior is not only a mathematically challenging problem but also has great practical significance.

In this paper we study a single degree of freedom nonlinear oscillator under combined parametric and forcing excitation

$$\ddot{x} - e(\mu - ax^2) \dot{x} + (\omega^2 + 2\epsilon \cos 2t) (y x - \epsilon \beta x^3) = \epsilon F \cos \omega t$$  \hspace{1cm} (1.1)

where \(0 < \epsilon \ll 1\), \(\mu\) is a positive or negative damping coefficient and \(\alpha, \beta, \gamma\) are positive or negative parameters.

Let \(x_1 = x, x_2 = \dot{x}\) system (1.1) takes the form

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\gamma \omega^2 x_1 + e[\mu x_2^2 + \beta \omega^2 x_1^3 - \alpha x_1 x_2^2 - 2(\gamma x_1 - \epsilon \beta x_1^3) \cos 2t + F \cos \omega t]
\end{cases}$$  \hspace{1cm} (1.2)

When linear damping and stiffness terms, parametric and forcing excitation of system (1.1) are
zeros, equation (1.2) becomes
\[
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = \varepsilon \beta \omega^2 x_1^3 - \varepsilon ax_1^3 x_2
\end{cases}
\tag{1.3}
\]

Because equation (1.3) is a degenerate system of codimension two, the linear part of the vector field is doubly degenerate
\[
A_0 = D_f(0, 0) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}
\tag{1.4}
\]

Mathematically, equation (1.2) can be regarded as universal unfolding with parametric and forcing perturbation of equation (1.3). Equation (1.2) can have codimension two bifurcations.

In Ref. [1] Holmes and Rand used the theory of differential dynamical systems to study the local and global bifurcations of van der Pol-Duffing strongly nonlinear oscillator. Greenspan and Holmes used Melnikov’s method to study the homoclinic bifurcations of van der Pol-Duffing strongly nonlinear oscillator with forcing excitation. Tang Jian-ning and Liu Zeng-rong used Melnikov’s method to study the complex bifurcations of 2-jet system and 3-jet system with universal unfolding. In the following we use Melnikov’s method to study heteroclinic orbit and subharmonic bifurcations and chaos of system (1.1).

II. Bifurcations to Heteroclinic Orbits

To study system (1.1) more conveniently, we rescale \( \beta \rightarrow \beta/\varepsilon \), so that equation (1.2) becomes
\[
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\gamma \omega^2 x_1 + \beta \omega^3 x_1^3 + \varepsilon [\mu x_2 - ax_1^3 x_2] \\
\quad -2 (\gamma x_1 - \beta x_1^3) \cos 2t + F \cos \Omega t
\end{cases}
\tag{2.1}
\]

When \( \varepsilon = 0 \), equation (2.1) becomes
\[
\begin{cases}
\dot{x}_1 = x_1 \\
\dot{x}_2 = -\gamma \omega^2 x_1 + \beta \omega^3 x_1^3
\end{cases}
\tag{2.2}
\]

Equation (2.2) is a Hamiltonian system, with Hamiltonian
\[
H = \frac{1}{2} x_1^2 + \frac{1}{2} \gamma \omega^2 x_1^3 - \frac{1}{4} \beta \omega^3 x_1^4 = h
\tag{2.3}
\]

where \( h \) is a constant.

Letting
\[
\begin{cases}
x_2 = 0 \\
-\gamma \omega^2 x_1 + \beta \omega^3 x_1^3 = 0
\end{cases}
\tag{2.4}
\]

we obtain: (I) When \( \beta \gamma > 0 \), there are three singular points at \((0, 0)\) and \((\pm \sqrt{\gamma/\beta}, 0)\).

(II) When \( \beta \gamma < 0 \), there is one singular point at \((0, 0)\).

In this paper we only study case (I). After making simple analysis of the singular points, we know easily

(i) when \( \gamma > 0 \), \((0, 0)\) is a center and \((\sqrt{\gamma/\beta}, 0)\) and \((- \sqrt{\gamma/\beta}, 0)\) are hyperbolic saddles.

(ii) when \( \gamma < 0 \), \((0, 0)\) is a hyperbolic saddle and \((\sqrt{\gamma/\beta}, 0)\) and \((- \sqrt{\gamma/\beta}, 0)\) are centers.