THEOREM OF THE STABILITY OF LINEAR NONAUTONOMOUS SYSTEMS UNDER THE FREQUENTLY-ACTING PERTURBATION AND ITS APPLICATION IN THE STABILITY ANALYSIS OF ROBOT

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Abstract

The necessary and sufficient condition of the stability of linear nonautonomous system under the frequently-acting perturbation has been given and proved on the basis of [1] and [2], and the theorem of the equivalence on the uniform and asymptotical stability in the sense of Liapunov and the stability under the frequently-acting perturbation of linear nonautonomous system has been given in this paper. Besides, the analysis of the dynamic stability of robot has been presented by applying the theorem in this paper, which is closer to reality.

Key words stability, nonautonomous system, frequently-acting perturbation, state transition matrix, robot

I. Introduction

In paper [1] the sufficient condition of the stability of linear nonautonomous system under the frequently-acting perturbation was given and proved. In paper [2], we expounded the key problem of the application of the theorem—calculation of the transition matrix and evaluation of its norm and induced another conclusion as follows. If the equilibrium point of the nonautonomous linear system is uniformly asymptotically stable in the sense of Liapunov, then the equilibrium point of the system is stable under the frequently-acting perturbation.

In this paper, the necessary condition of the stability of linear nonautonomous system under the frequently-acting perturbation has been proved, thus a more perfect theorem has been set up. Besides, the analysis of the stability of drive and control systems of a robot is presented by applying the theorem in this paper, which is closer to reality.

II. The Theorem of the Stability of Linear Nonautonomous Systems Under the Frequently-Acting Perturbation

Theorem 1 For a linear nonautonomous system

\[ x = A(t)x \]  \hspace{1cm} (2.1)
The origin (the equilibrium point) of the system (2.1) is stable under frequently-acting perturbation if and only if

\[ \sup_{t_0 \geq 0} \sup_{0 \leq t \leq t_0} \| \Phi(t, t_0) \|_1 \Delta m_0 < \infty \]  \hspace{1cm} (2.2)

\[ \| \Phi(t, t_0) \|_1 \to 0 \text{ as } t \to \infty \text{ uniformly in } t_0 \]  \hspace{1cm} (2.3)

or if there exist positive constants \( m \) and \( \lambda \), such that

\[ \| \Phi(t, t_0) \|_1 \leq m \exp[-\lambda(t-t_0)] \]  \hspace{1cm} (\forall t \geq t_0, \forall t_0) \hspace{1cm} (2.4)

where \( \Phi(t, t_0) \) is the state transition matrix, which is associated with \( A(t) \) and is the unique solution of the equation

\[ \frac{d}{dt} \Phi(t, t_0) = A(t) \Phi(t, t_0), \Phi(t_0, t_0) = 1 \]  \hspace{1cm} (2.5)

**Proof** The sufficient condition of the theorem has been proved in paper [1]. Then we're only proving its necessary condition next. To that end, it is necessary to expound briefly the definition of the stability of a general system under the frequently-acting perturbation. “Difference between the stability under the frequently-acting perturbation and stability in the sense of Liapunov is that the perturbation is not only given to the initial condition of motion but also to the differential equation of motion. If the differential equation of perturbed motion of unperturbed motion is given by

\[ \frac{dx_s}{dt} = X_s(t, x_1, \ldots, x_n) \hspace{1cm} (s = 1, 2, \ldots, n) \]  \hspace{1cm} (2.6)

suppose that the right-hand side of the equations is continuous in region

\[ t \geq 0, \hspace{0.5cm} |x_*| \leq ll \]  \hspace{1cm} (2.7)

and satisfies \( X(t, 0, \ldots, 0) = 0 \) and there exists and only solution under the given initial condition.

At the same time we examine equations

\[ \frac{dx_s}{dt} = X_s(t, x_1, \ldots, x_n) + R_s(t, x_1, \ldots, x_n) \]  \hspace{1cm} (2.8)

where function \( R_s \) defined in region (2.7) is determined by factor of frequently-acting perturbation, and is small enough, which possesses the condition of satisfying equation (2.8) having the only solution.”

Roughly speaking, the statement of unperturbed motion is stable under frequently-acting perturbation, which means that if any initial value \( x_i \) is very small and \( R \) due to perturbation is also very small, then value \( x_i \) always remains small enough at all times.

If equations (2.6) and (2.8) in matrix form, respectively

\[ \dot{x} = f(t, x) \]  \hspace{1cm} (2.9)

\[ \dot{x} = f(t, x) + R(t, x) \]  \hspace{1cm} (2.10)

Then the strict mathematical definition of the stability under the frequently-acting perturbation is as follows.