DYNAMIC ANALYSIS TO INFINITE BEAM UNDER A MOVING LINE LOAD WITH UNIFORM VELOCITY*

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Abstract

Based on the principle of linear superposition, this paper proves generalized Duhamel's integral which reverses moving dynamical load problem to fixed dynamical load problem. Laplace transform and Fourier transform are used to solve partial differential equation of infinite beam. The generalized Duhamel's integral and deflection impulse response function of the beam make it easy for us to obtain final solution of moving line load problem. Deep analyses indicate that the extreme value of dynamic response always lies in the center of the line load and travels with moving load at the same speed. Additionally, the authors also present definition of moving dynamic coefficient which reflects moving effect.

Key words generalized Duhamel's integral, integral transform, infinite beam, dynamic response, moving effect

I. Introduction

An infinite beam on elastic foundation not only can be looked on as dynamic model for a suspension bridge or a tension diagonal bridge11 but also can be used in dynamics analysis to a rail track, therefore, dynamic response of infinite beam under moving load has become the focus of discussion in past several decades. The problem also has been investigated by Timoshenko9, Fryba3, Steele4, Lee9, et al. We find, however, that all of the studies on this subject only discussed moving point load problem. Actually, either automobile load or a train load is a line distributed source but not a point one. In addition, main methods used in above-mentioned researches are eigen-function and the solution of the problem is always given as a form of series. For finite beam, the method is effective, but for infinite beam, there are infinite members, so it cannot construct solution accurately by using few summarization in series. This reason makes computation increase when we calculate numerical solution.

By means of integral transform and the principle of linear superposition, the paper deals with the dynamic response problem of infinite beam suffering with moving line load, which overcomes shortcomings generated by series expansion and the solution is analytic.

II. Mathematical Method

We assume that foundation of beam is Kelvin foundation which satisfies the following
two postulates:

(1) Foundation satisfies presumption of Winkler foundation, that is, opposition force of foundation is in proportion to deflection (Vertical displacement) of beam.

(2) There exists linear damping in the foundation, which is in proportion to ratio of variation of the deflection.

Under the two hypotheses, motion equation of Bernoulli-Euler beam on Kelvin foundation can be given by

$$EI \frac{d^4 u}{dx^4} + ku + c \frac{du}{dt} + m \frac{d^2 u}{dt^2} = F(x,t) \quad (2.1)$$

where $u = u(x,t)$ is the deflection of beam and $F(x,t)$ is the force acted on beam, $EI$ and $m$ are bending stiffness and mass in unit length of beam respectively, while $k$ and $c$ represent spring constant and damping constant of Kelvin foundation, respectively. For moving line load, it is described by the following formula

$$F(x,t; r_0) = P_0 \frac{H(r_0^2 - (x - vt)^2)}{2r_0} H(t) \quad (2.2)$$

in which $H(\cdot)$ is Heaviside step function while initial deflection and initial velocity of beam respectively satisfies their initial condition

$$u(x,t) \bigg|_{t=0} = 0, \quad \frac{du}{dt} \bigg|_{t=0} = 0 \quad (2.3)$$

Equations (2.1), (2.2), (2.3) form the complete mathematical model for definite solution problem.

III. Generalized Duhamel's Integral

It can be known from equation (2.1) that dynamic system of beam belongs to linear system. Considering a line load $P_0$ limited in range $[-r_0, r_0]$ forcelanding on beam at the time $t=0$, then it moving along direct $x$-axis, see Fig. 1. Define dynamic response of beam under line impulse load $F_d(x,t) = [H(r_0^2 - x^2)/2r_0] \delta(t)$ as deflection impulse response function (DIRF) of this linear system and denote it as $h_d(x,t)$. When time is $\tau$, the center of the line load lies on $x_\tau = vt$. In order to use the DIRF whose center of line load is lying on $x=0$ to express deflection the response of an arbitrary point $A$ at time $t$ caused by line impulse load at time $\tau$, we should convert coordinate.

Let original coordinate system be $xO$ and new coordinate system be $x'O'$ whose origin lies on the center of moving line load. Let $x_o$ and $x_o'$ represent coordinate in $xO$ and $x'O'$, respectively. Then the two coordinates satisfy

$$x_o' = x_o - vt \quad (3.1)$$

Therefore deflection response of point $A$ at time $t$ caused by line impulse load at time $\tau$ should be expressed as $h_d(x_o', t-\tau)$ in which $t-\tau$ is the time displacement. According to the principle of superposition of linear system, the deflection response of point $A$ at time $t$ caused by moving line load can be constructed by

![Fig. 1 Moving line load](image-url)