The Pulse oximeter is a noninvasive optical instrument which measures arterial oxygen saturation (SaO₂) in a pulsatile vascular bed such as the fingertip or earlobe. SaO₂ is defined as:

\[ \text{SaO}_2 = \frac{C_0}{C_0 + C_r} \times 100 \text{ per cent} \]  

where \( C_0 \) and \( C_r \) are the concentrations of oxygenated hemoglobin (HbO₂) and reduced hemoglobin (or deoxygenated hemoglobin) (Hb), in the blood sample. Pulse oximetry relies on the difference in absorption spectra between HbO₂ and Hb. Dual-wavelength measurements are usually made by placing a probe containing two light-emitting diodes (LEDs) whose spectral peaks are in the red and infra-red regions, and a photodiode on opposite sides of the vascular bed. The photodiode senses the light transmitted through the tissues, which comprises an alternating (AC) signal due to absorption by pulsatile arterial blood superimposed on a steady-state (DC) level arising from attenuation by venous blood, skin pigments and other nonpulsatile components. With most instruments, the DC level at each wavelength is used to normalise the corresponding AC signal amplitude. The ratio of these AC amplitudes at the two wavelengths has been shown to be a function of the arterial oxygen saturation, and this is the basis of the technique of pulse oximetry (WUKITSCH et al., 1988).

The first pulse oximeters were developed in Japan during the late 1970s. In this early work, whole blood was considered simply to be made up of Hb and HbO₂ solutions which obeyed the Beer-Lambert Law (YOSHIYA et al., 1980). This law describes the absorption of monochromatic light in the nonscattering solution:

\[ I = I_0 10^{-\varepsilon CD} \]  

where \( I_0 \) and \( I \) are the incident and transmitted light intensities through a cuvette of depth \( D \), containing a solution of concentration \( C \) and with an extinction coefficient \( \varepsilon \). Eqn. 2 may be rewritten in terms of optical density \( OD \) as:

\[ OD = \log \left( \frac{I_0}{I} \right) = \varepsilon CD \]  

Experience soon showed, however, that such a simple model gave a poor correlation between pulse oximeter readings and invasive measurements of arterial oxygen saturation (SHIMADA et al., 1984). For instance, at 80 per cent SaO₂ the pulse oximeter overestimated the saturation by approximately 5 per cent, an error which is clinically unacceptable.

As recognised by these early workers (SHIMADA et al., 1984), this discrepancy is due to light scattering phenomena. KRAMER et al. (1951), ANDERSON and SEKELJ (1967) and others had already conclusively shown that the Beer-Lambert law is not valid for whole blood, for which the optical density is nonlinear with respect to cuvette depth and haematocrit. In addition KRAMER et al. (1951) also reported that the optical density of whole blood is approximately 7–20 times greater than that of haemoglobin solutions. In whole blood, light scattering arises from the discontinuity in refractive index at the plasma/red blood
cell interface, the wavelength of light in the red and infra-red being of the same order of magnitude as the dimensions of a red blood cell. The high concentration of red blood cells in whole blood ensures that light scattered once is likely to be scattered again—hence the phenomenon of multiple scattering.

The complex optical properties of whole blood are such that, in practice, all commercial pulse oximeters are now empirically calibrated using procedures in which hypoxia is induced in normal subjects (SEVERINGHAUS and NAIFEH, 1987). As soon as steady-state hypoxia is reached, the values of the normalised red:infra-red ratio, of $SaO_2$, as measured from an arterial blood sample, are recorded. If such data are collected from a large number of volunteers, over a range of oxygen saturations, an empirical calibration curve can be obtained. The majority of such tests are carried out when the $SaO_2$ is greater than 70 per cent, the lowest tolerable steady-state hypoxia. Below this saturation, the calibration curve is largely estimated by extrapolation.

In an attempt to develop a model relating $SpO_2^*$ and dual-wavelength measurements on whole blood, we adopted a similar approach to that of STEINKE and SHEPHERD (1986). To study the relative importance of scattering and absorption, they measured the optical density of whole blood as a function of haematocrit and also the optical density of haemolysed blood. They then fitted a number of mathematical models for the optical properties of whole blood to their experimental data and found Twersky’s multiple scattering theory (TWERSKY, 1962; 1970a; b) to be the most appropriate. With the use of Twersky’s theory they investigated the role of light scattering in whole blood oximetry and concluded that scattering effects did not affect the linearity of whole blood oximeters. Instead, scattering effects were said to increase the sensitivity of these oximeters by contributing linearly to the total optical density change that occurs with altered oxygenation (STEINKE and SHEPHERD, 1986).

To extend this approach to pulse oximetry, we measured the optical density of whole and also of haemolysed blood as a function of cuvette depth. We chose the latter variable rather than haematocrit in view of the fact that pulse oximeters rely on arterial volume pulsations which are essentially changes in optical pathlength. We fitted three different theoretical models of optical transmission in whole blood to our experimental data to determine which model might be the most applicable for the pulsatile case. The model which gave us the closest fit was then used to obtain a relationship between $SaO_2$ and our dual-wavelength measurements.

1.1 Optical theories of whole blood

Among the many theories proposed to study the optical properties of whole blood, three have found general acceptance: Zdrojkowski and Pisharoty’s approach based on photon diffusion theory, Twersky’s analytical multiple scattering theory, and the Kubelka-Munk theory. All three approaches are now reviewed in the context of their applicability to pulse oximetry.

1.2 Photon diffusion theory

Photon diffusion theory, also known as radiative transfer theory, is based on observations of the transport characteristics of wave intensities. It relies on a differential equation called the equation of transfer which is equivalent to the Boltzmann equation used in the kinetic theory of gases (ISHIMARU, 1978; JOHNSON, 1970). Zdrojkowski and Pisharoty (1970) solved the diffusion equation for a collimated light beam incident on a slab of blood incorporating the concept of mean optical pathlength of photons in dense scattering media previously derived by SHOCKLEY (1962). Zdrojkowski and Pisharoty derived the following optical density equation of whole blood:

$$OD = -\log \left[ e^{-bd} + \frac{Q}{SB^2 - Q^2} \sinh (Qd) \right] \left[ 1 - e^{-bd} \left( \cosh (Qd) + \frac{B}{Q} \sinh (Qd) \right) \right]$$

(4)

where

$$b = \frac{1}{L} + \frac{1}{S}$$

(5)

$$Q = \frac{3}{2} \sqrt{\frac{B}{L}}$$

(6)

$$S = \frac{S_0(\lambda)}{H(H - 1)}$$

(7)

$$L = \log \left( \frac{e}{C_{in}} \right) \times 1.64$$

(8)

and $L$ is the mean optical path travelled by a photon before absorption, $S$ is the mean optical path before scattering, $e$ is the extinction coefficient in litre mmol$^{-1}$ cm$^{-1}$, $C_{in}$ is the concentration of haemoglobin in the whole blood sample in g d$^{-1}$, $H$ the fractional haematocrit and $d$ the cuvette depth.

1.3 Twersky’s analytical multiple scattering theory

Twersky’s analytical multiple scattering theory of light by large low-refracting and absorbing particles is based on electromagnetic wave theory (TWERSKY, 1962; 1970a; b). Twersky started by first considering the scattering and absorption effects of a single particle, and then introduced the interaction of many particles and statistical averages to finally arrive at an analytical solution to the multiple scattering problem. ANDERSON and SEKELJ (1967), LIPOWSKY et al. (1980) and STEINKE and SHEPHERD (1986) have shown this theory to be applicable to whole blood. Twersky’s equation for the variation of optical density is as follows:

$$OD = \log \left( \frac{I_0}{I} \right) = \varepsilon CD$$

$$- \log \left[ (1 - q)1 \times 10^{-\beta} + q1 \times 10^{-\delta} \right]$$

(9)

where

$$\beta = aDH(1 - H)$$

(10)

$$\delta = \frac{2q'maDH(1 - H)}{2m + aD(1 - H)}$$

(11)

$$m = \frac{\varepsilon CD}{H}$$

(12)

and

$\varepsilon = \text{constant depending on particle size, wavelength, photodiode aperture and refractive indices of plasma and red blood cells}$